

Math 143
Midterm 1
October 11, 2018

Name: _____

UR ID: _____

Circle your Instructor's Name:

Kalyani Madhu

Ian Alevy

PLEASE COPY THE HONOR PLEDGE AND SIGN:

(Cursive is not required).

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

YOUR SIGNATURE: _____

IMPORTANT: This exam has 8 questions, but you only need to answer 7. Below, circle the question you wish to **skip**. Then put an X on the question or in the answer box if that question has one.

1 2 3 4 5 6 7 8

1. (8 points)

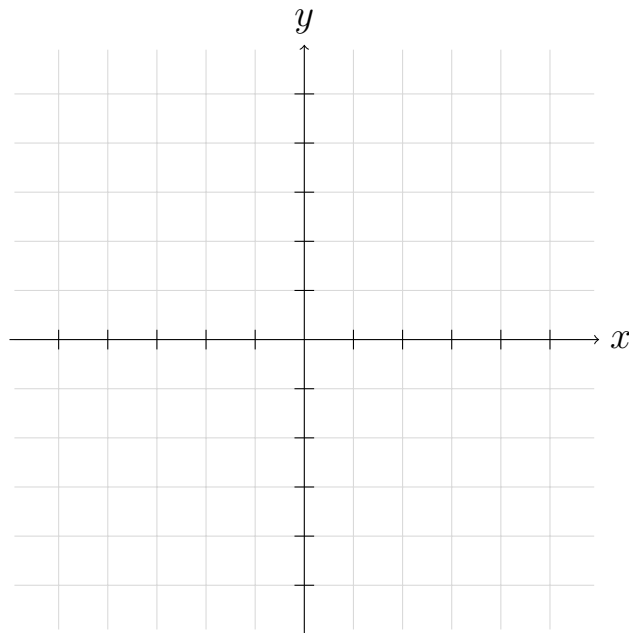
Two particles move in the plane. The first moves along the curve C_1 defined by the parametric equations

$$x = t, \quad y = t^2, \quad -2 < t < 2$$

and the second moves along the curve C_2 defined by the parametric equations

$$x = t - 2, \quad y = t, \quad -2 < t < 2$$

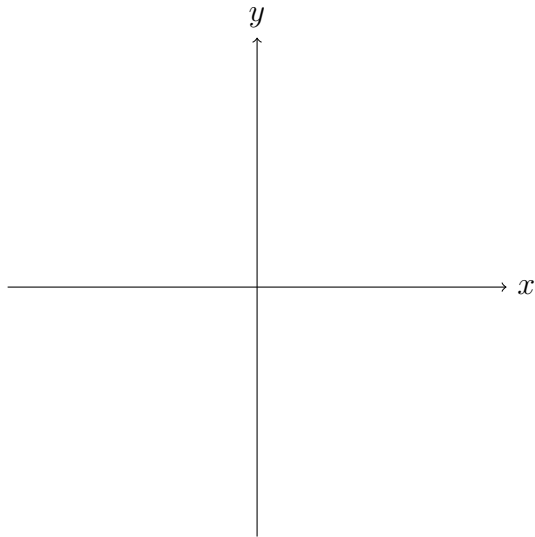
- (a) Plot both curves. Label them C_1 and C_2 . For both curves label the initial and terminal points, and provide arrows to show in which direction the curve is being sketched.



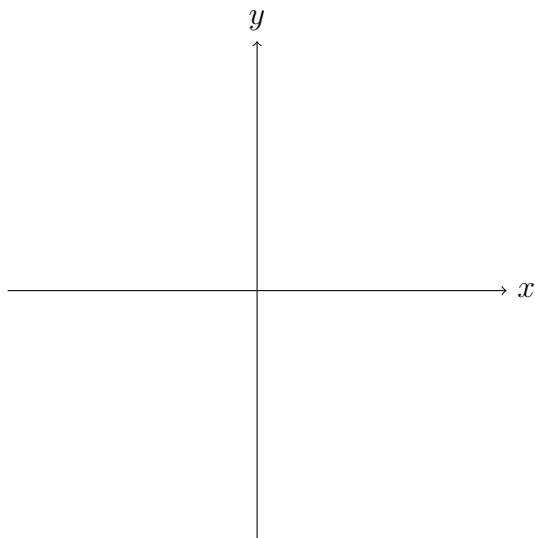
- (b) Do the particles collide? At what time or times? Justify your answer.

2. (8 points) Sketch the region of the plane whose polar coordinates satisfy the given conditions

(a) $2 \leq r \leq 3$ and $0 \leq \theta \leq \frac{\pi}{2}$



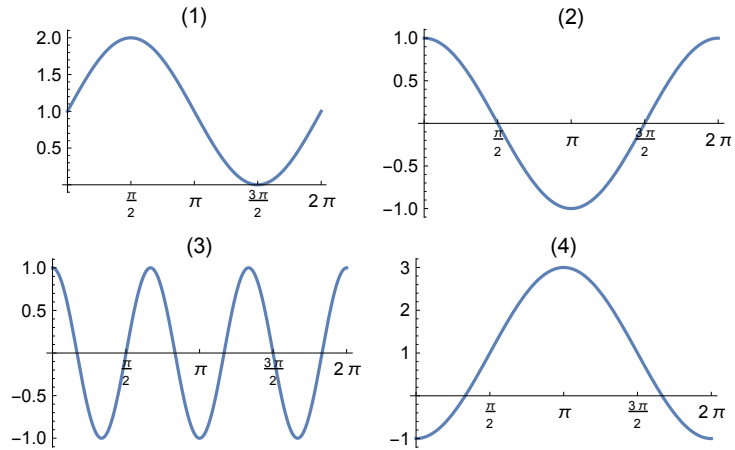
(b) $-2 < r < -1$ and $0 \leq \theta \leq \frac{3\pi}{4}$



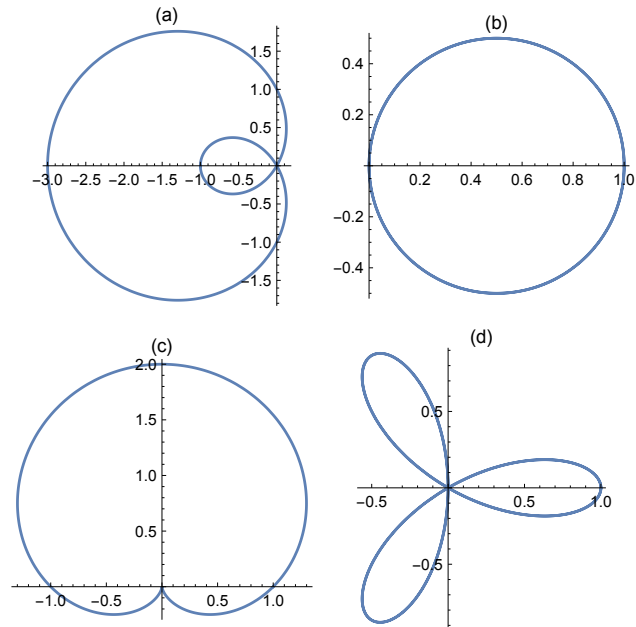
3. (8 points)

Match the graphs of functions sketched in the $r\theta$ plane to their polar plots in the xy -plane.

$r\theta$ -plane:



xy -plane:



Answers:

1.) _____

2.) _____

3.) _____

4.) _____

4. (8 points)

- (a) Set up, but do not evaluate, an integral that gives the area of the region that lies inside the polar curve $r = 2 \sin \theta$ and outside the polar curve $r = 1$.

Answer:

- (b) Find the slope of the tangent line to the polar curve $r = 2 \sin \theta$ when $\theta = \pi/6$.

Answer:

5. (8 points) (This question has 4 parts and is on two pages.)
Determine whether each sequence converges or diverges. If it converges, find its limit. Justify your answers.

(a) $\left\{ \frac{\sin(4n + 5) + 7n}{5n} \right\}_{n=1}^{\infty}$

Answer:

(b) $\left\{ \frac{\ln(n)}{\ln(2n)} \right\}_{n=1}^{\infty}$

Answer:

Question (5), second page

(c) $\left\{ \frac{\arctan(n)}{n} \right\}_{n=1}^{\infty}$

Answer:

(d) $\{\sin(\pi n)\}_{n=1}^{\infty}$

Answer:

6. (8 points) (This question has two parts and is on two pages.)

(a) A sequence $\{a_n\}$ is defined recursively by the conditions

$$a_1 = 1 \quad a_2 = 1 \quad a_3 = 2 \quad a_n = 2a_{n-1} + a_{n-3} \quad \text{for } n \geq 4.$$

Find the terms $\{a_3, a_4, a_5, a_6, a_7\}$ in the sequence.

Answer:

(Question (6), second page)

(b) Is the sequence

$$a_n = \frac{n}{2^n}$$

monotonic for $n \geq 2$? If it is, is it increasing or decreasing? Justify your answer.

7. (8 points)

(a) Suppose an infinite series $\sum_{n=1}^{\infty} a_n$ satisfies $\lim_{n \rightarrow \infty} S_n = 2$, where

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^n a_i.$$

i.) Find $\lim_{n \rightarrow \infty} a_n$, or state that there is not enough information.

ii.) Find the value of the sum $\sum_{n=1}^{\infty} a_n$, or state that there is not enough information.

(b) Suppose a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies $\lim_{n \rightarrow \infty} a_n = 1$

i.) Does $\{a_n\}_{n=1}^{\infty}$ converge?

ii.) Does $\sum a_n$ converge?

8. (8 points) Determine whether or not the following series converge or diverge. If the series converges, provide its sum. Show your work.

(a)
$$\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^{n+1}}$$

Answer:

(b)
$$\sum_{n=0}^{\infty} (e^n - e^{n+1})$$

Answer:

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.