

Math 143

Final

December 16, 2018

Name: _____

UR ID: _____

Circle your Instructor's Name:

Kalyani Madhu

Ian Alevy

PLEASE COPY THE HONOR PLEDGE AND SIGN:

(Cursive is not required).

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

YOUR SIGNATURE: _____

Part A

1. (12 points)

A curve is described by the parametric equations

$$x = e^t \cos(t), \quad y = e^t \sin(t), \quad 0 \leq t \leq 2\pi.$$

- (a) Compute $\frac{dy}{dx}$ as a function of t .

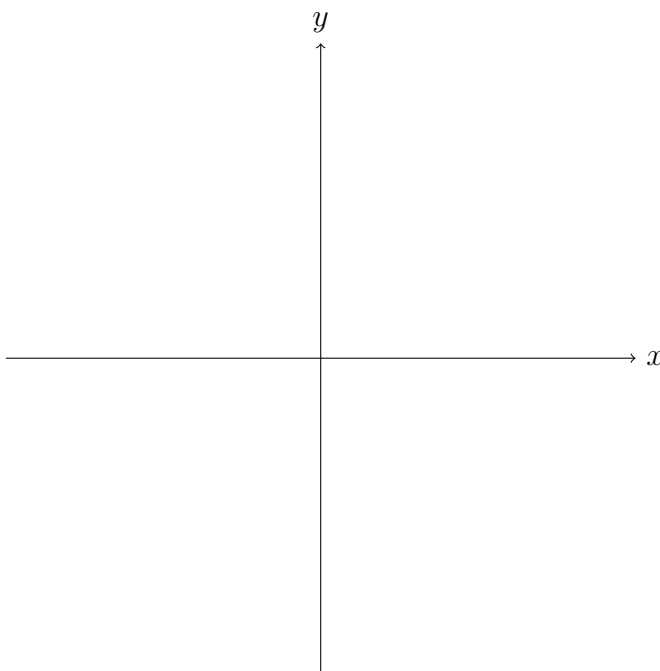
Answer:

- (b) For which t -values in the range $[0, 2\pi]$ is the tangent line horizontal?

Answer:

- (c) Set up, but do not evaluate, an integral which computes the length of the curve.

2. (7 points) On the axes below sketch the region of the plane whose polar coordinates satisfy the conditions $0 \leq \theta \leq 2\pi/3$ and $-2 \leq r \leq 0$.



3. (10 points) Find the area enclosed by one loop of the polar curve $r = \sin 2\theta$. You may need to use the following trigonometric identity:

$$\sin^2(2\theta) = \frac{1 - \cos 4\theta}{2}.$$

Answer:

4. (16 points) Determine whether each sequence converges or diverges. If it converges, check the appropriate box and write the limit in the space provided. If it diverges, check that box. **No work or justification is required, and no partial credit will be offered.**

(a) $\left\{ \sqrt[n]{n^2 + 2} \right\}_{n=1}^{\infty}$

Converges to limit _____

Diverges

(b) $\left\{ \frac{n!}{n(n-1)} \right\}_{n=1}^{\infty}$

Converges to limit _____

Diverges

(c) $\left\{ 3 \cos\left(\frac{\pi}{2n}\right) + 3 \tan\left(\frac{\pi}{2n}\right) \right\}_{n=1}^{\infty}$

Converges to limit _____

Diverges

(d) $\left\{ \frac{2^{n+1}}{3 + 7^n} \right\}_{n=1}^{\infty}$

Converges to limit _____

Diverges

5. (20 points) For the following series, determine if the series is a geometric series, telescoping series, alternating series, p -series, or none of these. Check **all** categories which apply to the given series. Next, determine whether or not the series converges conditionally, converges absolutely, or diverges. **No work or justification is required, and no partial credit will be offered.**

(a) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{2n+1}$

Check ALL that apply:

- Geometric Telescoping Alternating p -series None of these

Check one:

- Converges absolutely Converges conditionally Diverges
-

(b) $\sum_{n=1}^{\infty} (-0.75)^n$

Check ALL that apply:

- Geometric Telescoping Alternating p -series None of these

Check one:

- Converges absolutely Converges conditionally Diverges
-

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Check ALL that apply:

- Geometric Telescoping Alternating p -series None of these

Check one:

- Converges absolutely Converges conditionally Diverges

(d) $\sum_{n=1}^{\infty} \frac{2^{2n}}{3^{n+1}}$

Check ALL that apply:

- Geometric Telescoping Alternating p -series None of these

Check one:

- Converges absolutely Converges conditionally Diverges
-

(e) $\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n}$

Check ALL that apply:

- Geometric Telescoping Alternating p -series None of these

Check one:

- Converges absolutely Converges conditionally Diverges

6. (15 points)

(a) (10 pts) Use the integral test to show the convergence of

$$\sum_{n=1}^{\infty} ke^{-k}.$$

(You do not need to check the hypotheses of the test.)

(b) (5 pts) If we use

$$S_4 = \frac{1}{e} + \frac{2}{e^2} + \frac{3}{e^3} + \frac{4}{e^4}$$

to estimate the sum of this series, what is the upper bound on the error **given by the integral test**?

Answer: $R_4 \leq$

7. (10 points)

- (a) Use the limit comparison test to determine if the following series converges or diverges. (Show your work and justify your answer.)

$$\sum_{n=3}^{\infty} \frac{\sqrt[3]{n-2}}{4n+7}$$

Answer:

- (b) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3(2^n)}{7^{n+1}}.$$

Answer:

8. (10 points) Determine whether or not each of the following series converges or diverges. If it converges, determine if it converges conditionally or absolutely. Show your work, and say what test you used.

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{(n^2 + 2)^{n+2}}{(n!)^n}$$

Answer:

(b)
$$\sum_{n=0}^{\infty} \frac{2^{n+2}((2n)!)}{n!((2n-1)!)}$$

Answer:

Part B

1. (20 points) Find the first four non-zero terms of the power series centered at 0 for the following functions and its radius of convergence.

(a) $f(x) = \frac{2}{4 - 3x^2}$

Power series:	Radius:
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(b) $f(x) = \ln(x + 2)$

Power series:	Radius:
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2. (20 points)

- (a) Find the first 4 non-zero terms of the Taylor series for $\sin x$ at $\pi/6$.

(b) Find the first 4 non-zero terms of the Maclaurin series for $\sqrt{1+2x}$.

3. (30 points) Find the sum of the following series.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n4^n}$$

Answer:

(b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

Answer:

(c)
$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{4^n (2n)!}$$

Answer:

4. (30 points) Consider the function

$$f(x) = \frac{\sin x - x + \frac{1}{6}x^3}{2x^5}.$$

(a) Find the first 3 non-zero terms of a power series for the function.

(b) Find the first 3 non-zero terms for a series that represents $\int_0^1 f(x)dx$.

(c) Find $\lim_{x \rightarrow 0} f(x)$.

EXTRA SPACE. Use this space if you run out elsewhere. Be sure to label your problems and also include a note on the original page telling the graders to look for your work here.