

Math 143: Calculus III

Midterm II

November 21st, 2017

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Please circle your section:

Tucker TR 2:00pm

Yamazaki MW 9:00am

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- The use of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Put your answers in the spaces provided.
- You are responsible for checking that this exam has all 8 pages.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	20	
4	20	
5	20	
6	10	
7	10	
TOTAL	100	

1. (10 points) Determine whether the following series converges absolutely, converges only conditionally, or diverges. *Name any test you use.*

$$\sum_{n=1}^{\infty} \frac{2n \cos(n)}{n^3 + 7}$$

2. (10 points) Determine whether the following series converges absolutely, converges only conditionally, or diverges. *Name any test you use.*

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+10}$$

3. (20 points) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-3)^n (x+1)^n}{2^n \sqrt{n}}.$$

4. (20 points) Consider the function $f(x) = e^{-x}$.

(a) Find a power series expansion of $f(x)$ about $x = -4$. Write out the first three nonzero terms, and express the series in sigma notation.

(b) Use the ratio test to find the radius and interval of convergence of the series you found in (a). *No credit will be given for solutions not using the ratio test.*

5. (20 points)

(a) Find the Maclaurin series expansion of the function

$$f(x) = \frac{x^2 - \sin(x^2)}{x^6}.$$

Write out the first four nonzero terms, and express the series in sigma notation.

(b) What is the value of $f^{(12)}(0)$?

(c) What is the value of $f^{(11)}(0)$?

(d) What is the value of $\lim_{x \rightarrow 0} f(x)$?

6. (10 points) Write out the first two terms and then find the sum of each of the following convergent series. *You do not need to show the series are convergent. Your table of Maclaurin series expansions might be helpful.*

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(-3)^n} =$$

$$(b) \sum_{n=0}^{\infty} \frac{4^n}{(-5)^n n!} =$$

$$(c) \sum_{n=0}^{\infty} \frac{3(-1)^{n-1}}{(2n+1)2^{2n+1}} =$$

7. (10 points) Consider the parametric equations for a curve $C(\theta)$ defined by

$$x = 5 \cos(\theta), \quad y = 2 \sin(\theta)$$

(a) Eliminate the parameter, and write the resulting Cartesian equation in the form given below. *No credit will be given for solutions not showing any work.*

$$\frac{y^2}{4} =$$

(b) Find an interval of θ -values so that $C(\theta) = (5 \cos(\theta), 2 \sin(\theta))$ traces out the upper half of an ellipse (in the counter-clockwise direction).