NAME (please print legibly): __________________________________________
Your University ID Number: __________________________________________
Your University email ________________________________________________
Please circle your section:

Tucker TR 2:00pm          Yamazaki MW 9:00am

Pledge of Honesty
I affirm that I will not give or receive any unauthorized help on this exam and that all work
will be my own.

Signature: __________________________________________

• The use of calculators, cell phones, iPods and other electronic devices at
  this exam is strictly forbidden.

• Show your work and justify your answers. You may not receive full credit
  for a correct answer if insufficient work is shown or insufficient justification
  is given.

• Put your answers in the spaces provided.

• You are responsible for checking that this exam has all 8 pages.

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<thead>
<tr>
<th>QUESTION</th>
<th>VALUE</th>
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1. (10 points) Determine whether the following series converges absolutely, converges only conditionally, or diverges. Name any test you use.

\[ \sum_{n=1}^{\infty} \frac{2n \cos(n)}{n^3 + 7} \]
2. (10 points) Determine whether the following series converges absolutely, converges only conditionally, or diverges. Name any test you use.

\[
\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n + 10}
\]
3. (20 points) Find the radius and interval of convergence of the power series

\[ \sum_{n=1}^{\infty} \frac{(-3)^n(x + 1)^n}{2^n \sqrt{n}}. \]
4. (20 points) Consider the function $f(x) = e^{-x}$.

(a) Find a power series expansion of $f(x)$ about $x = -4$. Write out the first three nonzero terms, and express the series in sigma notation.

(b) Use the ratio test to find the radius and interval of convergence of the series you found in (a). *No credit will be given for solutions not using the ratio test.*
5. (20 points)

(a) Find the Maclaurin series expansion of the function

\[ f(x) = \frac{x^2 - \sin(x^2)}{x^6}. \]

Write out the first four nonzero terms, and express the series in sigma notation.

(b) What is the value of \( f^{(12)}(0) \)?

(c) What is the value of \( f^{(11)}(0) \)?

(d) What is the value of \( \lim_{x \to 0} f(x) \)?
6. (10 points) Write out the first two terms and then find the sum of each of the following convergent series. You do not need to show the series are convergent. Your table of Maclaurin series expansions might be helpful.

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(-3)^n} = \]

(b) \[ \sum_{n=0}^{\infty} \frac{4^n}{(-5)^n n!} = \]

(c) \[ \sum_{n=0}^{\infty} \frac{3(-1)^{n-1}}{(2n+1)2^{2n+1}} = \]
7. (10 points) Consider the parametric equations for a curve $C(\theta)$ defined by

$$x = 5 \cos(\theta), \quad y = 2 \sin(\theta)$$

(a) Eliminate the parameter, and write the resulting Cartesian equation in the form given below. No credit will be given for solutions not showing any work.

\[
\frac{y^2}{4} =
\]

(b) Find an interval of $\theta$-values so that $C(\theta) = (5 \cos(\theta), 2 \sin(\theta))$ traces out the upper half of an ellipse (in the counter-clockwise direction).