Math 143: Calculus III

Midterm II

November 8th, 2016

Please circle your section:

Yamazaki MWF 9am **Tucker TR 2pm**

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Your University email ____________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ____________________

- **The use of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the spaces provided.**
- **You are responsible for checking that this exam has all 8 pages.**

QUESTION	VALUE	SCORE
1	15	
2	20	
3	15	
4	15	
5	20	
6	15	
TOTAL	100	

Common Taylor series centered at $x = 0$:

Function	Taylor Series	Initial Terms	Converges for
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	$1 + x + x^2 + x^3 + x^4 + \cdots$	$-1 < x < 1$
$\frac{1}{1+x}$	$\sum_{n=0}^{\infty} (-1)^n x^n$	$1 - x + x^2 - x^3 + x^4 - \cdots$	$-1 < x < 1$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$	All x
$\sin(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	All x
$\cos(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	All x
$\tan^{-1}(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$-1 \leq x \leq 1$
$\ln(1+x)$	$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$-1 < x \leq 1$

1. (15 points) Use the root test or ratio test to determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(a)

$$\sum_{n=1}^{\infty} \frac{(-5)^n}{26^n n^2}$$

(b)

$$\sum_{n=1}^{\infty} \left(\frac{-3n^2 + n + 5}{2n^2 + 2} \right)^n$$

2. (20 points) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+7}$$

(b)

$$\sum_{n=1}^{\infty} \left(\frac{26}{5}\right)^n \frac{1}{n!}$$

3. (15 points) Consider the power series

$$\sum_{n=1}^{\infty} \frac{(9x-2)^n}{2^n \sqrt{n}}.$$

(a) Find its radius of convergence.

(b) Find its interval of convergence.

4. (15 points)

(a) Find a power series expansion of the function $f(x) = \dfrac{2x^4}{3 - 4x}$ about $x = 0$, write out the first five nonzero terms, and express the series in sigma notation.

(b) What are the radius and interval of convergence of the series you found in (a)?

5. (20 points)

(a) Find the Taylor series expansion of the function $f(x) = \ln(x)$ about $x = 2$, and fill in the blanks below.

$$\ln(2)+\underline{\hspace{3cm}}+\underline{\hspace{3cm}}+\underline{\hspace{3cm}}+\underline{\hspace{3cm}}+\cdots = \ln(2)+\sum_{n=1}^{\infty}\underline{\hspace{5cm}}$$

(b) What are the radius and interval of convergence of the series you found in (a)?

6. (15 points)

(a) Find the Maclaurin series expansion of the function $f(x) = 2x\sin(2x^2)$, write out the first five nonzero terms, and express the series in sigma notation.

(b) What are the radius and interval of convergence of the series you found in (a)?