Math 143: Calculus III

Final Exam
December 17th, 2016

Please circle your section:

Yamazaki MWF 9am  Tucker TR 2pm

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________
Your University email ________________________________

Pledge of Honesty
I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.
Signature: ____________________________________________

- The use of calculators, cell phones, and other electronic devices at this exam is strictly forbidden.

- You are responsible for checking that this exam has all 13 pages.

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Part A

1. (10 points) If a sequence below converges, find its limit, and justify by citing any theorems/rules you use. If a sequence below diverges, state whether it diverges because it oscillates, diverges to $+\infty$, or diverges to $-\infty$.

(a) $a_n = \left(\frac{\cos(n)}{n}\right)^2$

(b) $a_n = (-e)^n$

(c) $a_n = \frac{-2e^n + \sqrt{n}}{e^n + 1}$

(d) $a_n = \ln\left(\frac{n}{n^2 + 1}\right)$
2. (10 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{n \ln(n)^2} \]

(b) \[ \sum_{n=1}^{\infty} \left( \ln(4n^2 + 3n + 2) - \ln(4n^2 + 5n + 6) \right)^n \]
3. **(10 points)** Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a) 
\[ \sum_{n=1}^{\infty} \frac{n4^n + \sqrt{n}}{3^n - 7} \]

(b) 
\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(3^n)} \]
4. **(10 points)** Find the radius and interval of convergence of the power series below.

(a) \[ \sum_{n=1}^{\infty} \frac{3^n(2n^2 + 1)(x - 3)^n}{2^n(2n)!}. \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n(x + 3)^n}{4^n \sqrt{n}}. \]
5. (10 points) Consider the function \( f(x) = \frac{2}{(1 - 2x)^2} \).

(a) Write out the first five nonzero terms, and express in sigma notation a power series expansion for \( f(x) \) about \( x = 0 \).

(b) What are the radius and interval of convergence of the series you found in (a)?
6. (10 points) Consider the function \( f(x) = \ln(2x) \).

(a) Write out the first five nonzero terms, and express in sigma notation the Taylor series expansion for \( f(x) \) about \( x = 3 \).

\[
\ln(6) + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots = \ln(6) + \sum_{n=1}^{\infty} \frac{1}{n!}
\]

(b) What are the radius and interval of convergence of the series you found in (a)?
Part B

7. (10 points) Find the sum of the following convergent series. You do not need to justify that they converge.

(a) $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$

(b) $\sum_{n=1}^{\infty} \left[ \sin \left( \frac{1}{n} \right) - \sin \left( \frac{1}{n + 1} \right) \right]$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n (4)^{2n}}{3^{2n} (2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n \tau^{2n+1}}{(2n + 1) 8^{2n+1}}$
8. **(10 points)** Consider the function \( f(x) = \frac{3x - \sin(3x)}{x^3} \).

(a) Find the first five nonzero terms of the Taylor series expansion of \( f(x) \) about \( x = 0 \).

(b) What is the value of \( f^{(5)}(0) \)?

(c) What is the value of \( f^{(6)}(0) \)?

(d) What is the value of \( \lim_{x \to 0} f(x) \)?

(e) What is the Taylor polynomial of degree 4 of \( f(x) \) at \( x = 0 \)?
9. (10 points) Consider the parametric curve defined by

\[ x = t^2 \]

\[ y = t^3 - t. \]

(a) For which values of \( t \) does the curve have a horizontal tangent line?

(b) For which values of \( t \) does the curve have a vertical tangent line?

(c) Find the tangent line at \( t = 2 \).

(d) Determine intervals of \( t \)-values for which the parametric curve is concave up and intervals for which it is concave down.
10. \textbf{(10 points)} Consider the parametric curve defined by
\[ x = 2 \cos(t) \]
\[ y = 3 \sin(t). \]

(a) Sketch this curve on the graph above, indicating the direction of increasing \( t \).

(b) Fill in the area under the curve from \( t = \frac{\pi}{4} \) to \( t = \frac{\pi}{2} \) on your sketch above.

(c) Find the area under the curve from \( t = \frac{\pi}{4} \) to \( t = \frac{\pi}{2} \) using an appropriate integral.
11. (10 points) Consider the polar curve defined by $r = 1 + 2 \sin(\theta)$.

(a) Draw a clear sketch of the curve above.

(b) At which angles does the curve cross itself?

(c) Write down but do not evaluate an integral that would give the arc length of the curve from $t = 0$ to $t = 2\pi$. 

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12. (10 points) Consider the polar curve defined by $r = 1 + 2 \cos(\theta)$. Find the area inside the larger loop, but outside the smaller loop of this curve.