Math 143: Calculus III

Final Exam

December 17th, 2016

Please circle your section:

Yamazaki MWF 9am **Tucker TR 2pm**

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Your University email ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ______________________________

- **The use of calculators, cell phones, and other electronic devices at this exam is strictly forbidden.**
- **You are responsible for checking that this exam has all 13 pages.**

Part A		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	60	

Part B		
QUESTION	VALUE	SCORE
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
TOTAL	60	

Part A

1. (10 points) If a sequence below converges, find its limit, and justify by citing any theorems/rules you use. If a sequence below diverges, state whether it diverges because it oscillates, diverges to $+\infty$, or diverges to $-\infty$.

(a) $a_n = \left(\frac{\cos(n)}{n}\right)^2$

(b) $a_n = (-e)^n$

(c) $a_n = \frac{-2e^n + \sqrt{n}}{e^n + 1}$

(d) $a_n = \ln\left(\frac{n}{n^2+1}\right)$

2. (10 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=1}^{\infty} \frac{1}{n \ln(n)^2}$$

(b)

$$\sum_{n=1}^{\infty} \big(\ln(4n^2 + 3n + 2) - \ln(4n^2 + 5n + 6)\big)^n$$

3. (10 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=1}^{\infty} \frac{n4^n + \sqrt{n}}{3^n - 7}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(3^n)}$$

4. (10 points) Find the radius and interval of convergence of the power series below.

(a) $$\sum_{n=1}^{\infty} \frac{3^n(2n^2+1)(x-3)^n}{2^n(2n)!}.$$

(b) $$\sum_{n=1}^{\infty} \frac{(-1)^n(x+3)^n}{4^n\sqrt{n}}.$$

5. (10 points) Consider the function $f(x) = \dfrac{2}{(1-2x)^2}$.

(a) Write out the first five nonzero terms, and express in sigma notation a power series expansion for $f(x)$ about $x = 0$.

(b) What are the radius and interval of convergence of the series you found in (a)?

6. (10 points) Consider the function $f(x) = \ln(2x)$.

(a) Write out the first five nonzero terms, and express in sigma notation the Taylor series expansion for $f(x)$ about $x = 3$.

$$\ln(6) + \underline{\hspace{3cm}} + \underline{\hspace{3cm}} + \underline{\hspace{3cm}} + \underline{\hspace{3cm}} + \cdots = \ln(6) + \sum_{n=1}^{\infty} \underline{\hspace{5cm}}$$

(b) What are the radius and interval of convergence of the series you found in (a)?

Part B

7. (10 points) Find the sum of the following convergent series. You do not need to justify that they converge.

(a) $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$

(b) $\displaystyle\sum_{n=1}^{\infty} \left[\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right) \right]$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (4)^{2n}}{3^{2n}(2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{(2n+1)8^{2n+1}}$

8. (10 points) Consider the function $f(x) = \dfrac{3x - \sin(3x)}{x^3}$.

(a) Find the first five nonzero terms of the Taylor series expansion of $f(x)$ about $x = 0$.

(b) What is the value of $f^{(5)}(0)$?

(c) What is the value of $f^{(6)}(0)$?

(d) What is the value of $\lim_{x \to 0} f(x)$?

(e) What is the Taylor polynomial of degree 4 of $f(x)$ at $x = 0$?

9. (10 points) Consider the parametric curve defined by

$$x = t^2$$

$$y = t^3 - t.$$

(a) For which values of t does the curve have a horizontal tangent line?

(b) For which values of t does the curve have a vertical tangent line?

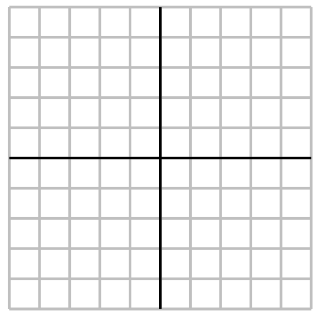
(c) Find the tangent line at $t = 2$.

(d) Determine intervals of t-values for which the parametric curve is concave up and intervals for which it is concave down.

10. (10 points) Consider the parametric curve defined by

$$x = 2\cos(t)$$

$$y = 3\sin(t).$$

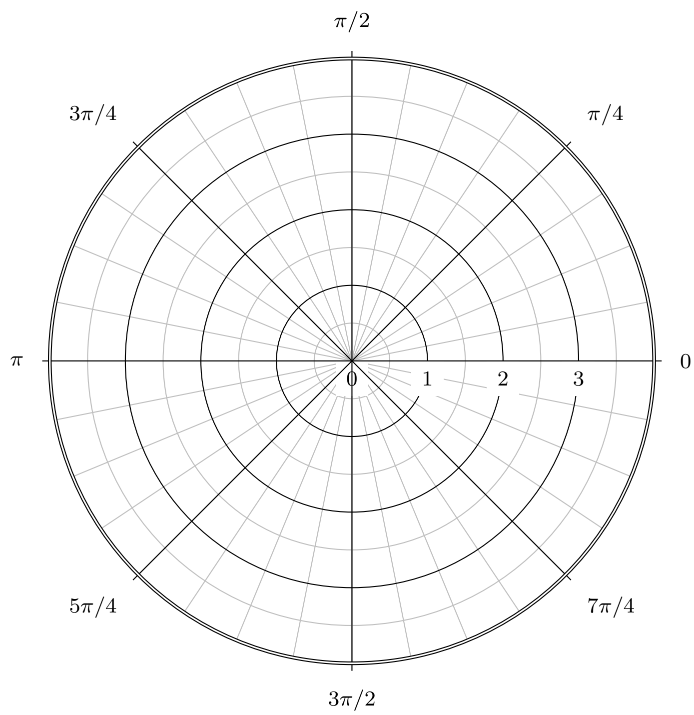


(a) Sketch this curve on the graph above, indicating the direction of increasing t.

(b) Fill in the area under the curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$ on your sketch above.

(c) Find the area under the curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$ using an appropriate integral .

11. (10 points) Consider the polar curve defined by $r = 1 + 2\sin(\theta)$.



(a) Draw a clear sketch of the curve above.

(b) At which angles does the curve cross itself?

(c) Write down but **do not evaluate** an integral that would give the arc length of the curve from $t = 0$ to $t = 2\pi$.

12. (10 points) Consider the polar curve defined by $r = 1 + 2\cos(\theta)$. Find the area inside the larger loop, but outside the smaller loop of this curve.