Math 143: Calculus III

Final Exam

May 8th, 2015

NAME (please print legibly): ____________________

Your University ID Number: ____________________

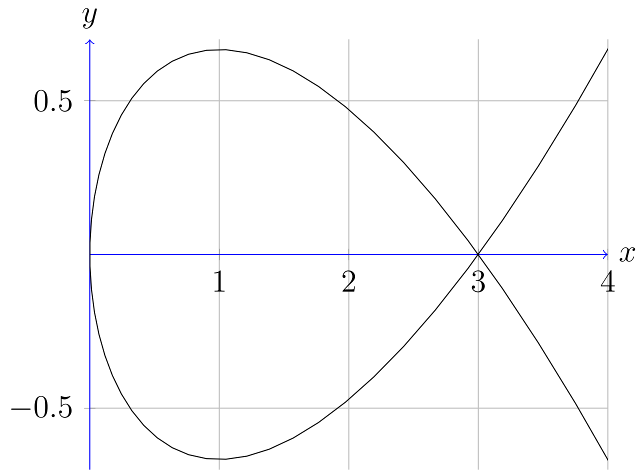
- **The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, and iPods.**
- **No notes, papers, books, or other aids are permitted, EXCEPT for the formula packet. Make sure that you have received a packet.**
- **Show work and justify all answers.** Box **final answers.**
- **You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Numerical or algebraic simplifications of answers are not required, EXCEPT when specifically stated otherwise.**
- **You are responsible for checking that this exam has all 13 pages.**

Part A		
QUESTION	VALUE	SCORE
1	12	
2	6	
3	12	
4	9	
5	8	
6	12	
7	7	
8	16	
9	18	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
10	20	
11	18	
12	10	
13	10	
14	18	
15	12	
16	12	
TOTAL	100	

Part A

1. (12 points) Consider the parametric curve $x = t^2$, $y = t - \frac{1}{3}t^3$, for $-2 \le t \le 2$, whose graph appears below.



(a) Find all points on the curve where the tangent line to the curve is horizontal or vertical.

(b) Find the arclength of the curve between $t = 0$ and $t = 1$.

2. (6 points) Make the following coordinate conversions. (Your answers should not have any functions in them.)

(a) Convert the point $(r, \theta) = (12, \pi/6)$ to rectangular coordinates.

(b) Convert the point $(x, y) = (3\sqrt{2}, 3\sqrt{6})$ to polar coordinates.

3. (12 points) Consider the polar curve $r = 2 + 2\cos(\theta)$.

(a) Sketch a rough plot of the curve, making sure to label any relevant distances on the x and y axes.

(b) Find the area enclosed by the curve.

4. (9 points) Let $z = 3 - 4i$ and $w = 6 + i$. Find the following in $a + bi$ form:

(a) $3z - 2w$.

(b) $\frac{w}{z}$.

(c) z^2.

5. (8 points) Find all complex numbers z such that $z^3 = 8i$. Express your answers in $a + bi$ form.

6. (12 points) Circle the correct response for each of the following (no work is required, and there is no partial credit):

(a) If $a_n = \tan^{-1}\left(\dfrac{n^2+2n+2}{n^3-3n+5}\right)$, then, as $n \to \infty$, the sequence a_n

(i) converges to the value 0.
(ii) converges to the value 1.
(iii) converges to the value π.
(iv) diverges to $+\infty$.
(v) diverges to $-\infty$.
(vi) diverges, but not to $+\infty$ or $-\infty$.

(b) Consider the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$. This series

(i) is a telescoping series, and converges to the value 1.
(ii) is a p-series, and converges to the value 5/6.
(iii) is a geometric series, and converges to the value 3/4.
(iv) is an alternating series, and converges to the value 0.
(v) diverges to $+\infty$ or $-\infty$.
(vi) diverges, but not to $+\infty$ or $-\infty$.

(c) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n$, where $a_n = \dfrac{\cos(n)}{2^n+n^2}$. This series is

(i) absolutely convergent, by the Comparison Test applied to $\displaystyle\sum_{n=1}^{\infty}|a_n| < \sum_{n=1}^{\infty}\frac{1}{2^n}$.
(ii) absolutely convergent, because it is a geometric series.
(iii) conditionally convergent, by the Alternating Series Test.
(iv) conditionally convergent, because it is a p-series.
(v) divergent, by the Ratio Test.
(vi) divergent, because its terms do not go to zero.

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7. (7 points) Use the Integral Test, Comparison Test, or Limit Comparison Test to determine whether $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$ converges to a finite value or diverges to $+\infty$.

8. (16 points) Use the Ratio or Root Test to determine whether each of these series converges to a finite value or diverges to $+\infty$.

(a) $\sum_{n=3}^{\infty} \left(\frac{3}{\ln n}\right)^n$.

(b) $\sum_{n=1}^{\infty} \frac{n!}{9^n}$.

9. (18 points) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. (Make sure to indicate CLEARLY any tests you use!)

(a) $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^n \sqrt{n}}{4^n}$.

(b) $\displaystyle\sum_{n=4}^{\infty} \frac{\cos(\pi n)}{n\ \ln(n)}$.

Part B

10. (20 points) Find the radius of convergence and the interval of convergence for each power series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3} \cdot (x-1)^n.$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n \sqrt{n}} \cdot x^n.$

11. (18 points) Find the requested Taylor polynomials:

(a) The degree-3 polynomial $T_3(x)$ for $f(x) = \dfrac{1}{1-x} \cdot \cos(x)$ centered at $x = 0$.

(b) The degree-2 polynomial $T_2(x)$ for $f(x) = \sqrt{4-3x}$ centered at $x = 1$.

12. (10 points) Find the Taylor series expansion for $f(x) = x^3 \tan^{-1}(2x^2)$ centered at $x = 0$. Make sure to give the general term of the expansion, NOT just the first few terms.

13. (10 points) Express the indefinite integral $\int \frac{\sin(x^3)}{x^2}\, dx$ as a power series.

14. (18 points) Let $f(x) = e^x$.

(a) Find $T_5(x)$, the degree-5 Taylor polynomial for $f(x)$, centered at $x = 0$.

(b) Find a value M such that $\left|f^{(6)}(x)\right| \leq M$ for all x in the interval $[-1, 1]$. (You can leave your answer in terms of e.)

(c) Using Taylor's Theorem, give an upper bound on the size of the remainder term $R_5(b) = |f(b) - T_5(b)|$ for b in the interval $[-1, 1]$.

15. (12 points) Circle the correct response for each of the following (no work is required, and there is no partial credit):

(a) The exact sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n \cdot n}$ is

(i) $\sin(-2)$.
(ii) $\tan^{-1}(1/2)$.
(iii) $\ln(3/2)$.
(iv) $\cos(1/2)$.
(v) $\sqrt{e}$.
(vi) $\sqrt{2}$.

(b) The limit $\displaystyle\lim_{x\to 0} \frac{\tan^{-1}(x) - x}{\sin(x) - x}$

(i) is equal to -3.
(ii) is equal to 0.
(iii) is equal to 1.
(iv) is equal to 2.
(v) is equal to 6.
(vi) diverges.

(c) Let $f(x) = \ln(1+x^3)$. The value of $f^{(9)}(0)$, the 9th derivative of $f(x)$ evaluated at $x = 0$, is equal to

(i) $9!$.
(ii) $9!/3$.
(iii) $9!/6!$.
(iv) 9.
(v) 3.
(vi) 1.

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16. (12 points) Circle the correct response for each of the following (no work is required, and there is no partial credit):

(a) The definite integral $\int_0^1 \cos(x^2)\,dx$ is equal to

(i) the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$.

(ii) the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n! \cdot (2n+2)}$.

(iii) the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)! \cdot (4n+1)}$.

(iv) the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+1)!}$.

(v) the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n)!}$.

(vi) $\sin(1)$, by direct integration.

(b) A possible Taylor polynomial at $x = 0$ for a function whose DERIVATIVE is e^{-x^2} is

(i) $1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$.

(ii) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$.

(iii) $1 - x^2 + \frac{x^4}{4}$.

(iv) $1 + x^2 + \frac{x^4}{4}$.

(v) $1 + x - \frac{x^3}{3} + \frac{x^5}{10}$.

(vi) $1 + x + \frac{x^3}{6} + \frac{x^5}{120}$.

(c) Power series and Taylor series

(i) are useless, have no applications, and are of interest only to mathematicians.

(ii) are quite useful and have a variety of practical applications in physics, chemistry, biology, computer science, engineering, economics, and other fields.
