Math 143: Calculus III

Final Exam

December 15th, 2015

Please circle your section:

Papadopulos MWF 9am **Tucker TR 2pm**

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- **The use of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the spaces provided.**
- **You are responsible for checking that this exam has all 14 pages.**

Part A		
QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
TOTAL	80	

Part B		
QUESTION	VALUE	SCORE
5	20	
6	20	
7	20	
8	20	
TOTAL	80	

Part C		
QUESTION	VALUE	SCORE
9	20	
10	20	
11	20	
12	20	
TOTAL	80	

Part A

1. (20 points) Determine whether the following sequences converge. If they converge find their limit. **Justify and show all your work.**

(a) $a_n = \sin\left(\frac{1}{n}\right)$

(b) $a_n = \dfrac{\cos^2(n)}{n}$

(c) $a_n = \dfrac{6n^7 + 5n + 3}{4n^7 + 2n + 8}$

(d) $a_n = \ln(n) - \ln(n^2 + 1)$

2. (20 points) Determine whether the following series converge or diverge and justify your answer. If they converge find their sum. **Justify and show all your work.**

(a)

$$\sum_{n=0}^{\infty} \left(\frac{-5}{4}\right)^n$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-3)^n + 5^{n-1}}{6^n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + n}$$

3. (20 points) Determine whether the following series converge or diverge and justify your answer. **Justify and show all your work.**

(a) Use the integral test to determine whether the following series converges or diverges. **To get full credit you must use the integral test.**

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

(b)

$$\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n+5} \right)^n$$

4. (20 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge. **Justify and show all your work.**

(a)

$$\sum_{n=2}^{\infty}(-1)^n\frac{n^2}{n^3-1}$$

(b)

$$\sum_{n=1}^{\infty}\frac{(-3)^n}{(3n-2)n!}$$

(c)

$$\sum_{n=1}^{\infty}(-1)^n\frac{n!}{2^n}$$

Part B

5. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

(a) Find a power series expansion of the function $f(x) = \dfrac{1}{1-3x}$ about $x = 0$, write out the first five nonzero terms, and express the series in sigma notation.

(b) What are the radius and interval of convergence of the series you found in (a)?

(c) Find a power series expansion of the function $f(x) = \dfrac{3}{(1-3x)^2}$ about $x = 0$, and express the series in sigma notation.

6. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

(a) Find the Taylor series expansion of the function $f(x) = \ln(2x)$ about $x = 3$, write out the first six nonzero terms, and express the series in sigma notation. (Hint: You should leave out the first term, $\ln(6)$, when you express it in sigma notation.)

(b) What are the radius and interval of convergence of the series you found in (a)?

7. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

(a) Find the Maclaurin series expansion of the function $f(x) = x^3 \sin(x/3)$, write out the first five nonzero terms, and express the series in sigma notation.

(b) What are the radius and interval of convergence of the series you found in (a)?

(c) Find the Taylor polynomials of degrees 8 and 10.

8. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

(a) Find the first five nonzero terms of the Maclaurin series expansion of the function

$$f(x) = \frac{\cos(x^2) - (1 - \frac{x^4}{2})}{x^8}.$$

(b) What is the value of $f^{(7)}(0)$?

(c) What is the value of $f^{(8)}(0)$?

(d) What is the value of $\lim_{x \to 0} f(x)$?

Part C

9. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

Consider the parametric curve defined by

$$x = t^2$$

$$y = t^3 - 3t.$$

(a) Calculate $\dfrac{dy}{dx}$.

(b) For which values of t does the curve have a horizontal tangent line?

(c) For which values of t does the curve have a vertical tangent line?

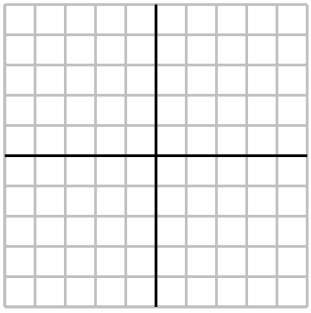
(d) Calculate $\dfrac{d^2y}{dx^2}$.

10. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

Consider the parametric curve defined by

$$x = \sin(t)$$

$$y = \cos(t).$$



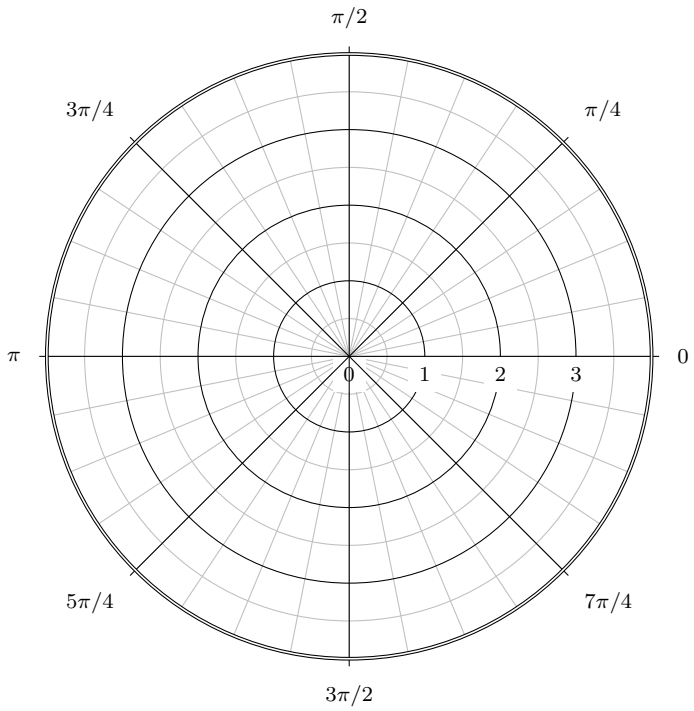
(a) Sketch this curve and fill in the area under the curve from $t = \frac{\pi}{6}$ to $t = \frac{\pi}{2}$ above.

(b) Find the area under the curve from $t = \frac{\pi}{6}$ to $t = \frac{\pi}{2}$ using an appropriate integral.

(c) Find the arclength of the entire curve using an appropriate integral.

11. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

Consider the polar curve defined by $r = 1 + 2\cos(\theta)$.



(a) Draw a clear sketch of the curve above.

(b) At which angles does the curve cross itself?

(c) Write down but **do not evaluate** an integral that would give the arclength of the curve from $t = 0$ to $t = 2\pi$.

12. (20 points) Justify and show all your work. No credit will be given if you do not justify and show all your work.

Consider the polar curve defined by $r = 2 - 2\sin(\theta)$. Find the area inside this curve.