Math 143: Calculus III
Midterm 2
November 13, 2008

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________

• The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

• Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• Please circle your simplified final answers, where applicable.

• You are responsible for checking that this exam has all 8 pages.

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1. (10 points) Consider the power series

\[ \sum_{n=1}^{\infty} (-1)^n \frac{3}{2^n \sqrt{n}} (x + 2)^n \]

(a) Find its radius of convergence.

(b) Find its interval of convergence.
2. (10 points) Consider the power series
\[ \sum_{n=1}^{\infty} \frac{n}{4!} (x - 1)^n \]

(a) Find its radius of convergence.

(b) Find its interval of convergence.
3. (15 points)

(a) Write $\frac{1}{1-x}$ as a power series centered at zero.

The radius of convergence for the series is: __________.

(b) Use (a) to find the power series centered at zero for $\frac{1}{4-x}$.

The radius of convergence for the series is: __________.

(c) Use (a) to find the power series centered at 3 for $\frac{1}{4-x}$.

The radius of convergence for the series is: __________.
4. **(15 points)** The power series representation for \( f(x) = \arctan x \), when \(|x| < 1\), is

\[
\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots
\]

Use this to find the power series representation for the following functions:

(a) \( 5 \arctan(3x^2) \)

The radius of convergence for the series is: ___________.

(b) \( \frac{1}{1 + x^2} \)

The radius of convergence for the series is: ___________.

(c) \( \frac{3x}{1 + 16x^4} \)

The radius of convergence for the series is: ___________.

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5. **(20 points)** This problem is about the Taylor series centered at \( a = \frac{\pi}{2} \) for the function \( f(x) = \sin x \).

(a) Find the first few terms of this series.

\[
\begin{align*}
c_0 &= \\
c_1 &= \\
c_2 &= \\
c_3 &= \\
c_4 &= \\
c_5 &= \\
c_6 &= 
\end{align*}
\]

(b) What is \( T_4(x) \)?

(c) Use \( T_4(x) \) to estimate \( \sin \left( \frac{7\pi}{12} \right) \). (Your answer should depend on \( \pi \).)

(d) Estimate the remainder \( R_4 \left( \frac{7\pi}{12} \right) \). (Your answer should depend on \( \pi \).)
6. (15 points) Let \( f(x) = e^x \).

(a) Find its Taylor series centered at zero. Show all your work!

(b) Use your answer in (a) to express \( \int e^{-x^2} \, dx \) as an infinite series.

(c) Use the first 2 terms in (b) to estimate \( \int_0^1 e^{-x^2} \, dx \)
7. (10 points) For which $x$ values is the function $f(x) = \cos x$ estimated by its 3rd degree Taylor polynomial centered at $a = 0$ with error at most $\frac{1}{100}$? (Hint: Use either Taylors Inequality or the Alternating Series Estimation Theorem. Your answer should be an interval.)

8. (5 points) Let $f(x) = 1 + 7(x - 3) + 29(x - 3)^{13} - 99(x - 3)^{100}$. Find the following:

\[ f(3) = \] 
\[ f''(3) = \] 
\[ f^{(100)}(3) = \]