# Math 143: Calculus III

Midterm 2

November 13, 2008

**NAME (please print legibly):** ______________________________

**Your University ID Number:** ______________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Please circle your simplified final answers, where applicable.**
- **You are responsible for checking that this exam has all 8 pages.**

| QUESTION | VALUE | SCORE |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 15 | |
| 4 | 15 | |
| 5 | 20 | |
| 6 | 15 | |
| 7 | 10 | |
| 8 | 5 | |
| TOTAL | 100 | |

**1. (10 points)** Consider the power series

$$\sum_{n=1}^{\infty}(-1)^n \frac{3}{2^n \sqrt{n}}(x+2)^n$$

(a) Find its radius of convergence.

(b) Find its interval of convergence.

**2. (10 points)** Consider the power series

$$\sum_{n=1}^{\infty} \frac{n}{4!}(x-1)^n$$

(a) Find its radius of convergence.

(b) Find its interval of convergence.

**3. (15 points)**

(a) Write $\frac{1}{1-x}$ as a power series centered at zero.

The radius of convergence for the series is: ____________.

(b) Use (a) to find the power series **centered at zero** for $\frac{1}{4-x}$.

The radius of convergence for the series is: ____________.

(c) Use (a) to find the power series **centered at** 3 for $\frac{1}{4-x}$.

The radius of convergence for the series is: ____________.

**4. (15 points)** The power series representation for $f(x) = \arctan x$, when $|x| < 1$, is

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$$

Use this to find the power series representation for the following functions:

(a) $5 \arctan(3x^2)$

The radius of convergence for the series is: ____________.

(b) $\dfrac{1}{1+x^2}$

The radius of convergence for the series is: ____________.

(c) $\dfrac{3x}{1+16x^4}$

The radius of convergence for the series is: ____________.

**5. (20 points)** This problem is about the Taylor series centered at $a = \frac{\pi}{2}$ for the function $f(x) = \sin x$.

(a) Find the first few term of this series.

$c_0 =$ ______________

$c_1 =$ ______________

$c_2 =$ ______________

$c_3 =$ ______________

$c_4 =$ ______________

$c_5 =$ ______________

$c_6 =$ ______________

(b) What is $T_4(x)$?

(c) Use $T_4(x)$ to estimate $\sin\left(\frac{7\pi}{12}\right)$. (Your answer should depend on $\pi$.)

(d) Estimate the remainder $R_4\left(\frac{7\pi}{12}\right)$. (Your answer should depend on $\pi$.)

**6. (15 points)** Let $f(x) = e^x$.

(a) Find its Taylor series centered at zero. Show all your work!

(b) Use your answer in (a) to express $\int e^{-x^2}\,dx$ as an infinite series.

(c) Use the first 2 terms in (b) to estimate $\int_0^1 e^{-x^2}\,dx$

**7. (10 points)** For which $x$ values is the function $f(x) = \cos x$ estimated by its 3rd degree Taylor polynomial centered at $a = 0$ with error at most $\frac{1}{100}$ ? (Hint: Use either Taylors Inequality or the Alternating Series Estimation Theorem. Your answer should be an interval.)

**8. (5 points)** Let $f(x) = 1 + 7(x-3) + 29(x-3)^{13} - 99(x-3)^{100}$. Find the following:

$$f(3) = \underline{\hspace{3cm}}$$
$$f"(3) = \underline{\hspace{3cm}}$$
$$f^{(100)}(3) = \underline{\hspace{3cm}}$$