Math 143: Calculus III

Midterm 1

November 21, 2008

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Please circle your simplified final answers, where applicable.**
- **You are responsible for checking that this exam has all 9 pages.**

QUESTION	VALUE	SCORE
1	20	
2	10	
3	10	
4	20	
5	15	
6	10	
7	9	
8	6	
TOTAL	100	

1. (20 points) Find the limits of the following sequences:

(a) $\lim_{n\to\infty} 4\arctan(n)$

(b) $\lim_{n\to\infty} \sin(n)$

(c) $\lim_{n\to\infty} \frac{2n+1}{n^2+3}$

(d) $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{3n}$

2. (10 points) Express the following continued fraction as a rational number.

$$8.626262\ldots$$

If you can express it as the sum of two rational numbers, that's also OK.

3. (10 points) Find the sum of the following series.

$$\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+2)}$$

Hint: Use partial fractions.

4. (20 points) Consider the series $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n\ln(n)}$.

(a) Use the Alternating Series Test to check if the series is **convergent**.

The test implies (circle ONE): convergence divergence inconclusive.

(b) Use the Ratio Test to check if the series is **absolutely convergent**.

The test implies (circle ONE): absolute convergence divergence inconclusive.

(c) Still checking for absolute convergence, we look at $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$.

(i) Use the Limit Comparison Test to compare with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$.

The test implies (circle ONE): convergence divergence inconclusive.

(ii) Use the Integral Test.

The test implies (circle ONE): convergence divergence inconclusive.

Then based on the four tests above, the series $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\ln(n)}$ is (circle ONE):

absolutely convergent conditionally convergent divergent unknown.

5. (15 points) Test the following series for convergence. Justify your answer, making sure to **name the convergence tests** that you are using.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \ln(n)}{7\,n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^2+3}}$

6. (10 points) Consider the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2}$$

(a) Does this series absolutely converge, conditionally converge or diverge? Justify your answer.

(b) Suppose we approximate the series by taking the sum of the first n terms, up to and including $(-1)^n(1/n^2)$. What is the first value of n for which our error is less than or equal to $1/10^4$?

7. (9 points) Consider the power series centered at $a = 1$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2\pi}{n} (x-1)^n$$

Find its interval of convergence, that is, find all x for which the series converges.

8. (6 points)

(a) We know $\sum_{n=1}^{\infty} a_n = 2$. Find $\lim_{n \to \infty} a_n =$

(b) We know $\lim_{n \to \infty} a_n = 7$. Find $\lim_{n \to \infty} a_{n+2} =$