MTH 143

Final

Sunday, December 16, 2007

Show all work (each step/computation) to receive full credit. No calculators. The exam contains 11 problems. Make sure it is complete.

No.	VALUE	SCORE
1	20	
2	15	
3	15	
4	15	
5	20	
6	15	
7	15	
8	30	
9	20	
10	20	
11	15	
TOTAL	200	

NAME : ________________________________

SECTION : ________________________________

1. Consider the series

$$\sum_{n=3}^{\infty} \frac{2}{(n-2)^2} \ .$$

a) Use the integral test to determine if the series converges or diverges.

b) Use the limit comparison test to determine if the series converges or diverges.

2. Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \sqrt{n}\, x^n \, .$$

Be sure to test the left and right end of your interval for convergence.

3. Find the sum of the series or show that it diverges.

a)

$$\sum_{n=1}^{\infty}(e^{-n} - e^{-(n+1)})$$

b)

$$\sum_{n=1}^{\infty}\frac{3^{n-1}}{5^n}$$

4. Find the area of the surface obtained by rotating the curve

$$y = 1 + x^2$$

from $x = 0$ to $x = 1$ about the y-axis.

5. Let $f(x) = 8\sqrt{x}$.
a) Find $T_2(x)$, the second degree Taylor polynomial of $f(x)$ at $a = 4$.

b) Use Taylor's inequality to estimate the accuracy of the approximation $f(x) \approx T_2(x)$ for $4 \leq x \leq 4.1$.

6. Let $f(x) = \sin(x)$ and consider $T_3(x)$, the third degree Taylor polynomial of $f(x)$ at $a = 0$. Use the alternating series estimation theorem to find the best possible estimate on the error $R_3(x) = f(x) - T_3(x)$ over the interval $-1 \leq x \leq 1$.

7. At time t the position of a particle is given by

$$x = 4(\cos t + t \sin t) \qquad y = 4(\sin t - t \cos t).$$

How many units of distance does the particle travel as t varies $0 \leq t \leq 2$?

8. A curve is traced by the parametric equations

$$x = t^2 + 1, \qquad y = -2t^3 + 6t.$$

a) Find all points (x, y) on the curve where the tangent line is horizontal.

b) Find all points (x, y) on the curve where the tangent line is vertical.

8. c) Compute $\frac{d^2y}{dx^2}$ and determine if the curve is concave upward or concave downward when $t > 0$.

9. a) Describe the point $(x, y) = (-1, 1)$ in terms of polar coordinates (r, θ) with $-\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$.

b) The polar curve $r^2 = \frac{4}{\sin\theta\,\cos\theta}$ can be described using a Cartesian equation $y = f(x)$. Find $f(x)$.

10. Find the area of the region enclosed by one loop of $r = \sin(2\theta)$.

11. A circle C_1 has center at the origin and radius 4. A second circle C_2 has a diameter with one end at the origin and the other end at the point $(8, 0)$. C_1 and C_2 intersect at two points, find the polar coordinates of both of them.