Part A

1. (10 points) If a sequence below converges, find its limit, and justify by citing any theorems/rules you use. If a sequence below diverges, state whether it diverges because it oscillates, diverges to $+\infty$, or diverges to $-\infty$.

(a) $a_n = \left(\frac{\cos(n)}{n}\right)^2$

(b) $a_n = (-e)^n$

(c) $a_n = \frac{-2e^n + \sqrt{n}}{e^n + 1}$

(d) $a_n = \ln\left(\frac{n}{n^2 + 1}\right)$

2. (10 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)^2}$$

(b)

$$\sum_{n=1}^{\infty} \left(\ln(4n^2 + 3n + 2) - \ln(4n^2 + 5n + 6)\right)^n$$

3. (10 points) Determine whether the following series converge absolutely, converge only conditionally, or diverge, naming any tests you use, and justifying their use completely.

(a)

$$\sum_{n=1}^{\infty} \frac{n4^n + \sqrt{n}}{3^n - 7}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(3^n)}$$

4. (10 points) Find the radius and interval of convergence of the power series below.

(a) $$\sum_{n=1}^{\infty} \frac{3^n(2n^2+1)(x-3)^n}{2^n(2n)!}.$$

(b) $$\sum_{n=1}^{\infty} \frac{(-1)^n(x+3)^n}{4^n\sqrt{n}}.$$

5. (10 points) Consider the function $f(x) = \dfrac{2}{(1-2x)^2}$.

(a) Write out the first five nonzero terms, and express in sigma notation a power series expansion for $f(x)$ about $x = 0$.

(b) What are the radius and interval of convergence of the series you found in (a)?

6. (10 points) Consider the function $f(x) = \ln(2x)$.

(a) Write out the first five nonzero terms, and express in sigma notation the Taylor series expansion for $f(x)$ about $x = 3$.

$$\ln(6) + \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_ + \cdots = \ln(6) + \sum_{n=1}^{\infty} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(b) What are the radius and interval of convergence of the series you found in (a)?

7. (10 points) Find the sum of the following convergent series. You do not need to justify that they converge.

(a) $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$

(b) $\displaystyle\sum_{n=1}^{\infty} \left[\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right) \right]$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (4)^{2n}}{3^{2n}(2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{(2n+1)8^{2n+1}}$

8. (10 points) Consider the function $f(x) = \dfrac{3x - \sin(3x)}{x^3}$.

(a) Find the first five nonzero terms of the Taylor series expansion of $f(x)$ about $x = 0$.

(b) What is the value of $f^{(5)}(0)$?

(c) What is the value of $f^{(6)}(0)$?

(d) What is the value of $\lim_{x \to 0} f(x)$?

(e) What is the Taylor polynomial of degree 4 of $f(x)$ at $x = 0$?

Part B

9. (15 points) Consider the parametric curve defined by

$$x = t^2$$

$$y = t^3 - t.$$

(a) For which values of t does the curve have a horizontal tangent line?

(b) For which values of t does the curve have a vertical tangent line?

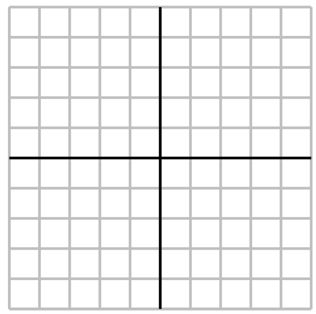
(c) Find the tangent line at $t = 2$.

(d) Determine intervals of t-values for which the parametric curve is concave up and intervals for which it is concave down.

10. (15 points) Consider the parametric curve defined by

$$x = 2\cos(t)$$

$$y = 3\sin(t).$$

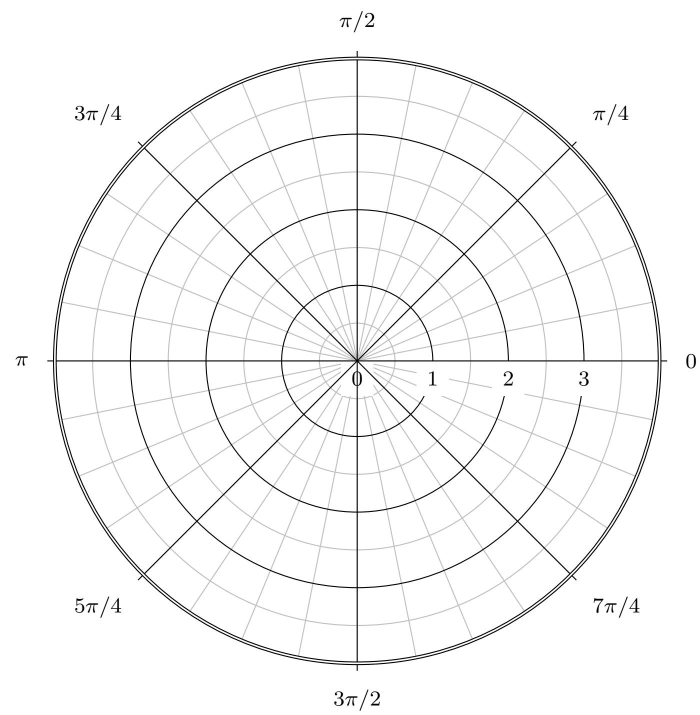


(a) Sketch this curve on the graph above, indicating the direction of increasing t.

(b) Fill in the area under the curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$ on your sketch above.

(c) Find the area under the curve from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$ using an appropriate integral .

11. (15 points) Consider the polar curve defined by $r = 1 + 2\sin(\theta)$.



(a) Draw a clear sketch of the curve above.

(b) At which angles does the curve cross itself?

(c) Write down but **do not evaluate** an integral that would give the arc length of the curve from $t = 0$ to $t = 2\pi$.

12. (15 points) Consider the polar curve defined by $r = 1 + 2\cos(\theta)$. Find the area inside the larger loop, but outside the smaller loop of this curve.