WARNING: these are just answers so you can check your work. They are NOT full solutions!!! You need to write full solutions on the exam. We'll go over these in the review session this coming week.

1. (10 points) Determine whether the following sequences converge or diverge. If they converge, find their limit. If they diverge, state whether they diverge to $+\infty$, $-\infty$ or because they oscillate. **Justify and show all your work.**

(a)

$$a_n = \cos(n)$$

DIV OSCL

(b)

$$a_n = \sin\left(\frac{1}{n}\right)$$

CONV to 0 by continuous function theorem

(c)

$$a_n = \ln(2n+3) - \ln(3n+2)$$

CONV to $\ln(2/3)$ by continuous function theorem

(d)

$$a_n = \cos\left(\frac{1}{n}\right)$$

CONV to 1 by continuous function theorem

(e)

$$a_n = \frac{\cos(n)}{n}$$

CONV to 0 by squeeze theorem

(f)

$$a_n = (-1)^n \frac{n}{(n+1)^2}$$

CONV to 0 by squeeze theorem

(g)

$$a_n = \left(\frac{9}{8}\right)^n$$

DIV to $+\infty$

(h)

$$a_n = \frac{(-1)^n}{6^n}$$

CONV to 0 by squeeze theorem

(i)

$$a_n = \ln\left(\frac{1}{n}\right)$$

DIV to $-\infty$

(j)

$$a_n = \frac{\ln(n)}{n}$$

CONV to 0 by l'Hopital's Rule

(k)

$$a_n = \frac{2^n}{n^2}$$

DIV to $+\infty$ by l'Hopital's Rule

(l)

$$a_n = \frac{n^2}{e^n - 1}$$

CONV to 0 by l'Hopital's Rule

(m)

$$a_n = \frac{\sin(n)}{e^n}$$

CONV to 0 by squeeze theorem

(n)

$$a_n = \frac{\sin(n)}{n!}$$

CONV to 0 by squeeze theorem

(o)

$$a_n = \ln\left(3n^2 + 1\right) - \ln\left(n + 1\right)$$

DIV to $+\infty$

2. (10 points) Determine whether the following series converge or diverge. If a series converges, find its sum. **Justify and show all your work. Name any test you are using.**

(a)

$$\sum_{n=0}^{\infty} \left(\frac{9}{8}\right)^n$$

DIV by GST

(b)

$$\sum_{n=0}^{\infty} \frac{(-3)^n + 3^n}{10^n}$$

CONV to 10/13 + 10/7

(c)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{6^n}$$

CONV to 6/7

(d)

$$\sum_{n=1}^{\infty} \left(\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right) \right)$$

CONV telescoping

(e)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

CONV to 1/3+1/6+1/9 (telescoping)

(f)

$$\sum_{n=2}^{\infty} \frac{3^n}{4^{2n+1}}$$

CONV to $9/(4^3 * 13)$

(g)

$$\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$$

DIV by CT to 1/n

3. (10 points) Determine whether the following series converge or diverge. **Justify and show all your work. Name any test you are using.**

(a)

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 5n}$$

CONV by CT

(b)

$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^{1.2} - 6}$$

CONV by LCT

(c)

$$\sum_{n=1}^{\infty} \frac{6^n + n}{5^n - 9}$$

DIV by LCT

(d)

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

DIV by test for div

(e)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + 8}$$

CONV by AST

(f)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$$

DIV by integral test

(g)

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$

CONV by CT

(h)

$$\sum_{n=1}^{\infty} \frac{n}{2n + 5}$$

DIV by test for divergence

(i)

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

CONV by AST

(j)

$$\sum_{n=1}^{\infty} \left(\frac{1}{n+1}\right)^2$$

CONV by CT or LCT

(k)

$$\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$$

DIV by test for DIV

(l)

$$\sum_{n=1}^{\infty} \frac{4}{n^{1.1}}$$

CONV by p-test and constant mult of conv is conv

(m)

$$\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$$

DIV by test for divergence

(n)

$$\sum_{n=1}^{\infty} \frac{1}{e^n}$$

CONV by GST

(o)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

CONV by CT or telescoping

(p)

$$\sum_{n=1}^{\infty} \frac{1}{n+1}$$

DIV by LCT

(q)

$$\sum_{n=1}^{\infty} e^{-n} - e^{-(n+1)}$$

CONV by GST, integral test, or telescoping

(r)

$$\sum_{n=1}^{\infty} \frac{n}{n^3+5n}$$

CONV by LCT

(s)

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

CONV By CT

4. (10 points) Use the integral test to determine whether the following series converges or diverges. **To get full credit you must use the integral test.**

(a)

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

DIV

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

CONV

(d)

$$\sum_{n=1}^{\infty} \frac{2n}{n^2+5}$$

DIV

5. (10 points) Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6}$.

(a) Use the Alternating Series Test to show the series converges.

alt $\frac{1}{n^6} > 0$ ✓
decr $\frac{1}{(n+1)^6} < \frac{1}{n^6}$ ✓
$\lim_{n\to\infty} \frac{1}{n^6} = 0$ ✓

(b) How many terms does it require to approximate the sum with error $\leq .001$?

$3^6 = 729 < 1000$ and $4^6 = 4096 > 1000$ so it takes 3 terms to approximate within .001.

(c) Approximate the sum of the series to within in .001. (Write it as a single fraction.)

$1 - \frac{1}{2^6} + \frac{1}{3^6} = \frac{46656}{46656} - \frac{729}{46656} + \frac{4096}{46656} = \frac{45991}{46656}$

(d) Give upper and lower bounds on the sum of the series.

$\frac{45991}{46656} - \frac{1}{1000} \leq s \leq \frac{45991}{46656} + \frac{1}{1000}$ so $0.984.... \leq s \leq 0.986...$

6. (20 points) Consider the series

$$\sum_{n=1}^{\infty} \left(\frac{-1}{5}\right)^n$$

(a) How many terms do you have to sum for the partial sum to be within $\frac{1}{125}$ of the convergent value of that series?
$5^3 = 125$ so it takes 2 terms to approximate within $\frac{1}{125}$.

(b) What is the approximation (partial sum) you get? (Write it as a single fraction.)
$\frac{-1}{5} + \frac{1}{25} = \frac{-4}{25}$

(c) What is the sum of series?
GST $\Longrightarrow \frac{-1/5}{1+1/5} = -\frac{1}{6}$

(d) How far off is your approximation from the actual sum?
$|-\frac{1}{6} - \frac{-4}{25}| = \frac{1}{150} \cong 0.00666...$