

1. (10 points) Determine whether the following series converges absolutely, converges only conditionally, or diverges. Name any test you use and justify its use.

1.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^{3/2}}{n^2 - 6}$$

2.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$$

**2. (10 points)** Determine whether the following series converges absolutely, converges only conditionally, or diverges. *Name any test you use and justify its use.*

1.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3 \ln(n)}{n}$$

2.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3 \ln(n)}{n^2}$$

3. (20 points) Find the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(-5)^n (x-3)^n}{(n-2)^{3/2} 4^n}.$$

**4. (20 points)**

(a) Consider the function  $f(x) = \ln(2x)$ . Find a power series expansion of  $f(x)$  about  $x = 3$ .

(b) Use the ratio test to find the radius and interval of convergence of the series you found in (a). *No credit will be given for solutions not using the ratio test.*

**5. (20 points)**

(a) Find the Maclaurin series expansion of the function

$$f(x) = \frac{2e^{\frac{x}{2}} - 2 - x}{x^2},$$

write out the first four nonzero terms, and express the series in sigma notation.

(b) What is the value of  $f^{(10)}(0)$ ?

(c) What is the value of  $f^{(11)}(0)$ ?

(d) What is the value of  $\lim_{x \rightarrow 0} f(x)$ ?

**6. (10 points)** Write out the first three terms and then find the sum of each of the following series. *Your table of Maclaurin series expansions might be helpful.*

$$(a) \sum_{n=0}^{\infty} \frac{1}{n! 3^n} 10^n =$$

$$(b) \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{(2n+1)} \frac{1}{2^{2n+1}} =$$

$$(c) \sum_{n=1}^{\infty} \frac{4^n (-1)^{n-1}}{n 5^n} =$$

**7. (10 points)**

Consider the parametric equations

$$x = \sin(\theta), \quad y = 1 + \sin^2(\theta)$$

- (a) Eliminate the parameter, and write the parametric equations in Cartesian form such that

$$y =$$

- (b) Find bounds for  $x$  and  $y$ .