1. (10 points) Determine whether the following sequences converge or diverge. If they converge, find their limit. If they diverge, state whether they diverge to $+\infty$, $-\infty$ or because they oscillate. **Justify and show all your work.**

(a)

$$a_n = \cos(n)$$

(b)

$$a_n = \sin\left(\frac{1}{n}\right)$$

(c)

$$a_n = \ln(2n+3) - \ln(3n+2)$$

(d)

$$a_n = \cos\left(\frac{1}{n}\right)$$

(e)

$$a_n = \frac{\cos(n)}{n}$$

(f)

$$a_n = (-1)^n \frac{n}{(n+1)^2}$$

(g)

$$a_n = \left(\frac{9}{8}\right)^n$$

(h)

$$a_n = \frac{(-1)^n}{6^n}$$

(i)

$$a_n = \ln\left(\frac{1}{n}\right)$$

(j)

$$a_n = \frac{\ln(n)}{n}$$

(k)

$$a_n = \frac{2^n}{n^2}$$

(l)

$$a_n = \frac{n^2}{e^n - 1}$$

(m)

$$a_n = \frac{\sin(n)}{e^n}$$

(n)

$$a_n = \frac{\sin(n)}{n!}$$

(o)

$$a_n = \ln\left(3n^2 + 1\right) - \ln\left(n + 1\right)$$

2. (10 points) Determine whether the following series converge or diverge. If a series converges, find its sum. **Justify and show all your work. Name any test you are using.**

(a)

$$\sum_{n=0}^{\infty} \left(\frac{9}{8}\right)^n$$

(b)

$$\sum_{n=0}^{\infty} \frac{(-3)^n + 3^n}{10^n}$$

(c)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{6^n}$$

(d)

$$\sum_{n=1}^{\infty} \left(\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right)\right)$$

(e)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

(f)

$$\sum_{n=2}^{\infty} \frac{3^n}{4^{2n+1}}$$

(g)

$$\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$$

3. (10 points) Determine whether the following series converge or diverge. **Justify and show all your work. Name any test you are using.**

(a)

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 5n}$$

(b)

$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^{1.2} - 6}$$

(c)

$$\sum_{n=1}^{\infty} \frac{6^n + n}{5^n - 9}$$

(d)

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(e)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + 8}$$

(f)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$$

(g)

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$

(h)

$$\sum_{n=1}^{\infty} \frac{n}{2n + 5}$$

(i)

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

(j)

$$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} \right)^2$$

(k)

$$\sum_{n=1}^{\infty} \ln \left(\frac{1}{n} \right)$$

(l)

$$\sum_{n=1}^{\infty} \frac{4}{n^{1.1}}$$

(m)

$$\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$$

(n)

$$\sum_{n=1}^{\infty} \frac{1}{e^n}$$

(o)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

(p)

$$\sum_{n=1}^{\infty} \frac{1}{n+1}$$

(q)

$$\sum_{n=1}^{\infty} e^{-n} - e^{-(n+1)}$$

(r)

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 5n}$$

(s)

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

4. (10 points) Use the integral test to determine whether the following series converges or diverges. **To get full credit you must use the integral test.**

(a)

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

(d)

$$\sum_{n=1}^{\infty} \frac{2n}{n^2+5}$$

5. (10 points) Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6}$.

(a) Use the Alternating Series Test to show the series converges.

(b) How many terms does it require to approximate the sum with error $\leq .001$?

(c) Approximate the sum of the series to within in .001. (Write it as a single fraction.)

(d) Give upper and lower bounds on the sum of the series.

6. (20 points) Consider the series

$$\sum_{n=1}^{\infty} \left(\frac{-1}{5} \right)^n$$

(a) How many terms do you have to sum for the partial sum to be within $\frac{1}{125}$ of the convergent value of that series?

(b) What is the approximation (partial sum) you get? (Write it as a single fraction.)

(c) What is the sum of series?

(d) How far off is your approximation from the actual sum?