**1. (10 points)** Determine whether the following series converges absolutely, converges only conditionally, or diverges. *Name any test you use and justify its use.*

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}\cos(n)}{n^3 - 2}$$

**2. (10 points)** Determine whether the following series converges absolutely, converges only conditionally, or diverges. *Name any test you use and justify its use.*

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$$

**3. (15 points)** Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-4)^n (x+1)^n}{9^n \sqrt{n}}.$$

**4. (15 points)** Consider the function $f(x) = e^{-x}$.

(a) Find the Taylor series of $f(x)$ about $x = 3$. Write out the first three nonzero terms, and express the series in sigma notation.

(b) Use the ratio test to find the radius and interval of convergence of the series you found in (a). *No credit will be given for solutions not using the ratio test.*

**5. (15 points)**

(a) Find the Maclaurin series expansion of the function

$$f(x) = \frac{\cos(x^2) - 1}{x^4}.$$

Write out the first four nonzero terms, and express the series in sigma notation.

(b) What is the value of $f^{(12)}(0)$?

(c) What is the value of $f^{(11)}(0)$?

(d) What is the value of $\lim_{x \to 0} f(x)$?

**6. (15 points)** Write out the first two terms and then find the sum of each of the following convergent series. *You do not need to show the series are convergent. Your table of Maclaurin series expansions might be helpful.*

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(-2)^n} =$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-5)^n}{2^n n!} =$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{5(-1)^{n-1}3^{2n+1}}{(2n+1)4^{2n+1}} =$

**7. (20 points)** Consider the parametric equations for a curve $C(\theta)$ defined for all $\theta$ by

$$x = 5\cos(2\theta), \qquad y = 2\sin(2\theta)$$

(a) Eliminate the parameter, and write the resulting Cartesian equation in the form given below. *No credit will be given for solutions not showing any work.*

$$\frac{y^2}{4} =$$

(b) Find parametric equations $x = f(t)$ and $y = g(t)$ for a circle of radius 2 centered at the origin together with an interval of $t$-values such that the circle is traced out once in the counterclockwise direction starting at $(x, y) = (2, 0)$ at $t = 0$.

Common Taylor series centered at $x = 0$:

| Function | Taylor Series | Initial Terms | Converges for |
|---|---|---|---|
| $\frac{1}{1-x}$ | $\sum_{n=0}^{\infty} x^n$ | $1 + x + x^2 + x^3 + x^4 + \cdots$ | $-1 < x < 1$ |
| $\frac{1}{1+x}$ | $\sum_{n=0}^{\infty} (-1)^n x^n$ | $1 - x + x^2 - x^3 + x^4 - \cdots$ | $-1 < x < 1$ |
| $e^x$ | $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ | $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ | All $x$ |
| $\sin(x)$ | $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ | $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ | All $x$ |
| $\cos(x)$ | $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ | $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ | All $x$ |
| $\tan^{-1}(x)$ | $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ | $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$ | $-1 \leq x \leq 1$ |
| $\ln(1+x)$ | $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$ | $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ | $-1 < x \leq 1$ |