

1. (10 points) Determine whether the following sequences converge or diverge. If they converge, find their limit. If they diverge, state whether they diverge to $+\infty$, $-\infty$ or because they oscillate. **Justify and show all your work.**

(a)

$$a_n = (-1)^n \sin\left(\frac{13}{n}\right)$$

(b)

$$a_n = (-4)^n$$

2. (10 points) Determine whether the following sequences converge or diverge. If they converge, find their limit. If they diverge, state whether they diverge to $+\infty$, $-\infty$ or because they oscillate. **Justify and show all your work.**

(a)

$$a_n = \frac{n^2}{e^{2n}}$$

(b)

$$a_n = \ln \left(\frac{7n - 7}{6n + 4} \right)$$

3. (10 points) Determine whether the following series converges or diverges. If it converges, find its sum. **Justify and show all your work. Name any test you are using.**

$$\sum_{n=0}^{\infty} \frac{3^n + 4^n}{3^{2n}}$$

4. (10 points) Consider the telescoping series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \left[\ln \left(\frac{1}{n} \right) - \ln \left(\frac{1}{n+1} \right) \right]$$

(a) Find the first three partial sums.

$$s_1 =$$

$$s_2 =$$

$$s_3 =$$

(b) Find a formula for the k^{th} partial sum $s_k = \sum_{n=1}^k \ln \left(\frac{1}{n} \right) - \ln \left(\frac{1}{n+1} \right)$.

(c) Determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, find its sum.

5. (10 points) Use the integral test to determine whether the following series converges or diverges. **To get full credit you must use the integral test.**

$$\sum_{n=1}^{\infty} 3n^2 e^{-n^3}$$

6. (10 points) Determine whether the following series converges or diverges. **Justify and show all your work. Name any test you are using.**

$$\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^2 - \sqrt{n}}$$

7. (10 points) Determine whether the following series converge or diverge. **Justify and show all your work. Name any test you are using.**

$$\sum_{n=1}^{\infty} 3^{1/n}$$

8. (10 points) Determine whether the following series converge or diverge. **Justify and show all your work. Name any test you are using.**

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

9. (10 points) Determine whether the following series converges or diverges. **Justify and show all your work. Name any test you are using.**

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n+5}$$

10. (10 points) Consider the convergent alternating series $\sum_{n=0}^{\infty} \left(\frac{-1}{4}\right)^n$.

(a) How many terms does it require to approximate the sum with error $\leq \frac{1}{64} = 0.015625$?

(b) Approximate the sum of the series to within in $\frac{1}{64}$. (Write it as a single fraction.)

(c) What is the sum of infinite series?