Warm-up:

1. Vocabulary : Discuss the meaning of the following sentence:

   An infinite series is said to converge to a sum, S, if and only if the limit, as n approaches infinity, of the partial sums of the series is equal to S.

2. Use sigma notation to describe the following series. Then come up with a different sigma expression for the same series.

   (a) $1 + \frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \dots$

   (b) $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

3. Write out all the terms of the following finite sums:

   (a) $\sum_{i=3}^{7} \frac{1+2^i}{37i^2}$

   (b) $\sum_{z=-3}^{1} \frac{z^2+2t}{e^z}$

4. For the following series, find S_3, the third partial sum.

$$\sum_{n=1}^{\infty} \frac{n^2+2^n}{3+n^2}$$

   Using only the tools we have so far, can you tell whether or not this series converges?

Problems:

1. Answer the following true or false:

   (a) If a series $\sum a_n$ satisfies $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

   (b) If a series $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

   (c) If $\{S_n\}$ is the sequence of partial sums of a series $\sum a_n$, and $\lim_{n\to\infty} S_n = 6$, then we are able to determine $\lim_{n\to\infty} a_n$.

   (d) If $\{S_n\}$ is the sequence of partial sums of a series $\sum a_n$, and $\lim_{n\to\infty} S_n = 6$, then $\sum a_n = 6$.

2. A **geometric** series has the form $a + ar + ar^2 + ar^3 + \ldots$. The number r is called the common ratio. For such a series, the nth partial sum is given by

$$S_n = \frac{a(1-r^{n+1})}{1-r}.$$

   (a) Go over the algebra we used in class to come up with the nth partial sum. Agree that a geometric series converges if and only if $|r| < 1$.

   (b) Determine the sum of the series $\displaystyle\sum_{n=2}^{\infty} 5(.89)^{n+1}$.

   (c) Determine the sum of the series $\displaystyle\sum_{n=3}^{\infty} \frac{3^{n-1}+1}{4^{n+1}}$.

   (d) The following series is an example of a power series:

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{4^n}.$$

   If we think of the power series as a function f from $\mathbb{R}$ to $\mathbb{R}$, what is $f(2)$? What is the domain of f?