Warm-up:

1. Plot the following polar points in the xy-plane.

   (a) $\left(1, \frac{\pi}{3}\right)$ (b) $\left(-1, \frac{\pi}{3}\right)$ (c) $\left(1, -\frac{\pi}{3}\right)$ (d) $\left(-1, -\frac{\pi}{3}\right)$

   For each point, find a second polar coordinate pair that describes it.

2. (a) Give polar coordinates to represent the given Cartesian (rectangular) points.

      i. $(1, 1)$ ii. $(-1, 1)$ iii. $(1, -1)$ iv. $(-1, -1)$

   (b) The Cartesian points above can be embedded in a circle. Give both the Cartesian equation for this circle and the polar equation for it.

Problems:

1. We will sketch the polar curve $r = \cos\theta$ for $0 \leq \theta \leq \pi$.

   (a) First graph it in the $r\theta$-plane. That is, draw a θ axis horizontally and an r axis vertically, and graph $r = \cos\theta$.

   (b) Now draw a pair of xy-axes.
   Sketch the curve $r = 2\cos\theta$ by using your graph in (a). Start by observing how the graph in (a) behaves on the interval $[0, \pi/2]$. Translate that behavior to polar coordinates in the xy-plane. Then move on to the interval $[\pi/2\pi]$...

   (c) Place arrows on your sketch to show in which direction the curve is being sketched as θ increases.

   (d) Now switch to rectangular coordinates, and check to see that your sketch makes sense. (You will need to complete the square.)

2. (a) Sketch the polar curve $r = 2\cos(2\theta)$ for $0 \leq \theta \leq 2\pi$ using the same technique. (You must do step (a)!) It's helpful to draw the dotted lines $\theta = \pi/4$ and $\theta = 3\pi/4$.

   (b) This time check your work using wolfram alpha.

   (c) While you're on wolfram alpha, use it to plot $r = \cos(n\theta)$ and $r = \sin(n\theta)$ for several values of n. Develop a conjecture about this family of curves that depends on whether it's cos or sin and whether n is odd or even. These curves are called polar roses. Polar roses are pretty.

3. Here is the equation of a polar curve called a limaçon:

$$r = 1 + 2\sin\theta; 0 \leq \theta \leq 2\pi.$$

Limaçons have the form $r = a + b\sin\theta$ and $r = a + b\cos\theta$. If $a < b$, the curve will develop a loop. If $a = b$, the limaçon will be a cardioid. Sketch the limaçon. When you do the $r\theta$-plane graph at the beginning, be sure to figure out the x-intercepts correctly. What dotted lines (as in 2a) might be helpful?

Now find the area bounded by the inner loop of the limaçon.