Workshop (11): Taylor and Maclaurin Series

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**Warmup:**

1. Let $f(x)$ have power series expansion

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n.$$

   (a) Find the first 5 derivatives of the power series.

   (b) Evaluate the first 5 derivatives of the power series at $a$.

   (c) Find each of $c_0, c_1, c_2, c_3, c_4$ in terms of derivatives of $f$ at $a$.

   (d) Generalize your result in (c) to the formula for the Taylor series of $f$ centered at $a$.

2. Suppose a function $f$ satisfies:

   (a) $f(2) = 6$

   (b) $f'(2) = 1$

   (c) $f''(2) = 3$

   (d) $f'''(2) = -2$

   (e) $f^{iv}(2) = 0$

   Give the first five terms of the Taylor series for $f$ centered at $x = 2$.

**Problems:** Before you begin, have someone write on the board the Maclaurin series for $e^x, \sin x, \arctan x, \ln(1-x)$, and $\cos x$.

1. Find power series for the following functions and their radii of convergence.

   (a) $g(x) = 3x^2 \sin(x/4)$

   (b) $h(x) = 6xe^{x^2}$

2. Evaluate the power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$

   (b) $\displaystyle\frac{8}{6} + \frac{16}{24} + \frac{32}{120} + \frac{64}{720} + \dots$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$

3. Use series to evaluate the limits

   (a) $\displaystyle\lim_{x\to 0} \frac{x - \tan^{-1} x}{x^3}$

   (b) $\displaystyle\lim_{x\to 0} \frac{1 - \cos x}{1 + x - e^x}$

4. (From MT2, Fall '16)

   (a) Use the Taylor series formula to develop a power series for $f(x) = \ln x$ about $x = 2$. Generate enough terms so that you can find a general term for the series in terms of $n$. (That is, write it as $\sum_{n=0}^{\infty} c_n (x-2)^n$ for a specific $c_n$.)

   (b) Now notice that $\ln x = \ln(2 + (x - 2))$. Use techniques from the previous chapter to find a power series for $\ln x$ about $x = 2$. (That is, find a power series for the derivative of $\ln(2+(x-2))$, and then integrate it.)