Warmup:

1. We can find power series for functions by using the geometric series. Recall that, if $|r| < 1$, then

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n.$$

   Use this fact to find a power series for

$$f(x) = \frac{1}{1-x}.$$

   What is the radius of convergence of the series?

2. We will need to differentiate and integrate series. Find the following:

   (a) $\dfrac{d}{dx}\left(\displaystyle\sum_{n=0}^{\infty}(-1)^{n-1}3^{n+1}5x^n\right)$

   (b) $\displaystyle\int\sum_{n=0}^{\infty}(-1)^{n-1}3^{n+1}5x^n dx$

3. Recall that differentiating and integrating a series does not affect the radius of convergence, but it can affect the interval of convergence. Find the radius and interval of convergence of

$$\sum_{n=0}^{\infty}\frac{(x)^n}{n^2}.$$

   Then take its derivative and find the interval of convergence of the result.

Problems: For each of the functions below, find a power series to represent it and its radius of convergence.

1. $f(x) = \dfrac{x^2}{3 - 2x}$
2. $g(t) = \ln(2 + t)$
3. $h(z) = \arctan(z)$ (Work this one out even if you already know the answer.
4. $r(x) = 10x^3 \arctan(2x)$ (Use (3).)
5. $f(x) = \dfrac{1}{(2 - 3x)^2}$