Warm-up:

1. Consider a curve C given by the parametric equations

$$x = f(t),\ y = g(t).$$

With your group, find the formulas:

(a) $\dfrac{dy}{dx} =$

(b) $\dfrac{d^2y}{dx^2} =$

(c) The area under the curve from $t = a$ to $t = b$.

(d) The arc length of the curve from $t = a$ to $t = b$.

How might you go about remembering these formulas?

Problems

1. Find parametric equations that give the following curve: A square whose corners are the points $(0,0), (1,0), (1,1)$, and $(0,1)$. Sketch this curve over the interval $0 < t \leq 4$ in a counter-clockwise direction. (You will have to take each side separately, with different parametrizations for $0 < t \leq 1, 1 < t \leq 2$, etc.)

2. Find a, b, c, d and k so that the parametric equations

$$x = a + b\sin(kt), y = c + d\cos(kt)$$

sketch a circle of radius 3 centered at $(1, -2)$ exactly once on the interval $0 \leq t \leq 4\pi/3$.

3. Let C_1 be the parametric curve with $x(t) = e^t \cos(t)$ and $y(t) = e^t \sin(t)$.

   (a) Think about how this curve behaves. How is it different from C_2 given by $x(t) = \cos(t)$ and $y(t) = \sin(t)$? Sketch this curve as t increases from 0 to 2π. What happens as $t \to \infty$?

   (b) Compute dy/dx for this curve.

   (c) Find an equation for the tangent line to C_1 at time $t = \pi/2$ and at time $t = \pi/6$.

   (d) Find two points on C_1 where the tangent line is horizontal, and two points where the tangent line is vertical.

   (e) Find an integral giving the arclength of the curve from $t = 0$ to $t = 2\pi$. (Do not evaluate the integral.) Find an integral giving the area underneath the curve from $t = \pi/4$ to $t = \pi/2$. (Do not evaluate the integral.) Sketch the region whose area you have found.