MTH142 Workshop 9: Integration by parts; Trigonometric Integrals

Warm-Up

1. Evaluate the following integrals:

(a) $\int \sin^3\theta\cos^4\theta d\theta$

(b) $\int \tan^3 x\sec x dx$

(c) $\int_0^\pi \cos^4(2t)dt$

(d) $\int \tan^2 x\sec^2 x dx$

2. Based on your work from number 1, why is it easier to solve $\int \sin^m x\cos^n x dx$ when either m or n is odd? What about $\int \tan^k x\sec^j x dx$ when either k is odd or j is even?

Problems

1. Solve the following integrals:

(a) $\int x\cos(5x)dx$

(b) $\int \arctan(4y)dy$

(c) $\int e^{2\theta}\sin(3\theta)d\theta$

2. (a) Use integration by parts to prove the reduction formula:

$$\int (\ln x)^n\, dx = x(\ln x)^n - n\int (\ln x)^{n-1}\, dx$$

(b) Use part (a) to solve $\int_1^e (\ln x)^3\, dx$.

3. First make a substitution and then use integration by parts to evaluate the integral:

(a) $\int e^{\sqrt{z}} dz$

(b) $\int \cos(\ln x)\, dx$

4. Consider the following integral.

$$\int \sin^2 x \cos^2 x dx$$

(a) Solve the integral by using the half angle formulas:

$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$
$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$

(b) Solve the integral by first recognizing that it is equivalent to $\int (\sin x \cos x)^2\, dx$. Then use the identity $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$.

5. (a) Prove the reduction formula:

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

Hint: Use integration by parts with $u = \cos^{n-1} x$ and $dv = \cos x dx$. When solving this, it may help to note that $\sin^2 x = 1 - \cos^2 x$.

(b) Use part (a) to evaluate $\int \cos^2 x dx$.

(c) Use parts (a) and (b) to evaluate $\int_0^{\pi/2} \cos^4 x dx$.