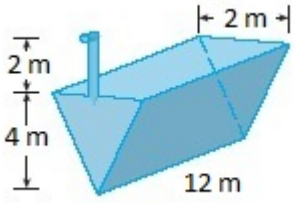
MTH142 Workshop 7: More Volumes

1. Find the volume of the tank shown below. Note: Ignore the pipe sticking up 2m above the top.

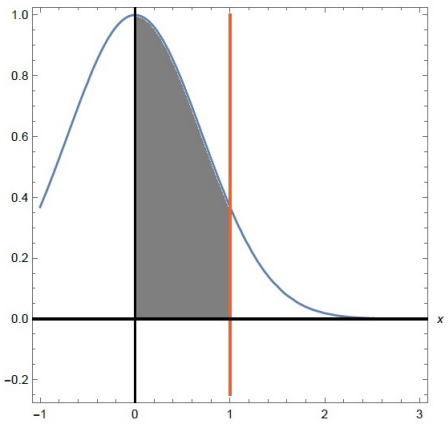


2. Use the Half Angle Formulas to find the volume of the solid obtained by rotating the region bounded by $y = \sin x$, $y = \cos x$, and $0 \leq x \leq \pi/4$; about the line $y = -1$. Sketch the region, the solid, and a typical washer.

3. Use the method of Cylindrical Shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the given line.

   (a) $xy = 1$, $x = 0$, $y = 1$, $y = 3$; about the x-axis

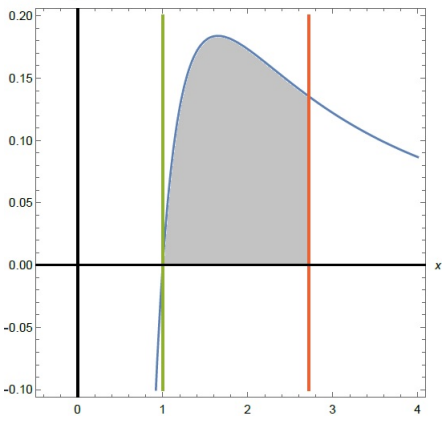
   (b) $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$; about the y-axis (the region is shaded below)



4. Graph the region bounded by the given curves. Find the volume of the solid obtained by rotating the region about the given line. You may use any method you want. (In making your choice, it may help to decide whether it is easier to integrate with respect to x or y.)

   (a) $x = y^2 - 1$, $y = -x + 1$; about $x = -1$

   (b) $y = \frac{\ln x}{x^2}$, $y = 0$, $x = 1$, $x = e$; about the y-axis (region shaded below)



5. Use cylindrical shells to find the volume of a solid torus (the donut-shaped solid shown in figure below) with radii r and R.
   [**Hint:** After you set up the integral, make the substitution $t = x - R$. The resulting integral should look familiar. You can solve it by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.]