MTH142 Workshop 5: Substitution

1. Integrate the following:

(a) $\displaystyle\int \frac{\sin\sqrt{a}}{\sqrt{a}}\,da$

(b) $\displaystyle\int \frac{dk}{xk+y} \quad (x \neq 0)$

(c) $\displaystyle\int_0^1 (3N-2)^{50}\,dN$

(d) $\displaystyle\int_1^2 \frac{e^{1/z}}{z^2}\,dz$

2. Recall the Half Angle Formulas:

$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2} \qquad \cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

Solve the following integral 3 ways, **(a)** once by taking $u = \sin y$, **(b)** once by taking $u = \cos y$, and **(c)** once by using the identity $2\sin y\cos y = \sin(2y)$. **(d)** Show that these are all equal up to constant by using the half angle formulas:

$$\int \sin y \cos y dy$$

3. Evaluate the following integrals by taking a substitution.:

(a) $\displaystyle\int \frac{dx}{(-3x+2)^4}$

(b) $\displaystyle\int \frac{dx}{-3x+2}$

(c) $\displaystyle\int \frac{e^x dx}{3e^{2x}+2}$

(d) $\displaystyle\int_1^2 \frac{t}{\sqrt{t-1}}dt$ [**Hint**: If $u = t-1$, what does t equal in terms of u?]

(e) $\displaystyle\int v^5\sqrt{1+v^3}dv$

(f) $\displaystyle\int \frac{du}{(1+\sqrt{u})^4}$

4. If f is continuous and $\int_0^4 f(z)dz = 10$, find $\int_0^2 f(2z)dz$.

5. Evaluate $\int_{-2}^{2} (x+3)\sqrt{4-x^2}dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

6. Throughout assume $f(x)$ is continuous on $[-a, a]$, $a > 0$.

(a) Use a substitution to show that if $f(x)$ is an odd function (i.e. $f(-x) = -f(x)$ for all x), then $\int_{-a}^{a} f(x)\ dx = 0$.

Hint: Consider $\int_{-a}^{0} f(x)\ dx$ with $u = -x$.

(b) Similarly show that if $f(x)$ is an even function (i.e. $f(-x) = f(x)$ for all x), then $\int_{-a}^{a} f(x)\ dx = 2\int_0^a f(x)\ dx$.