

MTH142 Workshop 4: The fundamental theorem of calc.; Indefinite integrals

1. Solve the following indefinite integrals:

$$(a) \int \frac{4x^3 + x^{1/3}}{6x^{4/3}} dx$$

$$(b) \int \frac{\sin x}{\cos^2 x} dx$$

$$(c) \int \csc x \tan x \cos x dx$$

2. Suppose

$$f(x) = \int_1^{e^x} \frac{1-t}{2+\ln t} dt$$

For what values of x does $f(x)$ have a local maximum?

3. Let

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } 1 < x \leq 2 \\ 0 & \text{if } x > 2 \end{cases}$$

and $g(x) = \int_0^x f(t) dt$.

(a) Find an expression for $g(x)$ similar to the one for $f(x)$.

(b) Sketch the graphs of f and g .

(c) Where is f differentiable? Where is g differentiable? [**Hint:** What is the derivative of g ?]

4. Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}$$

for all $x > 0$.

5. The acceleration function (in m/s^2) is $a(t) = 6t + 3$ and the initial velocity is $v(0) = -6$ m/s for particle moving along a line. Find:

(a) the velocity at time t .

(b) the distance (not displacement) traveled during the first 2 seconds.

6. (a) If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t) dt$ represent?
- (b) If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t , what does $\int_0^{120} r(t) dt$ represent?
- (c) If x is measured in meters and $f(x)$ is measured in newtons, what are the units for $\int_0^{100} f(x) dx$?
- (d) If the units for x are feet and the units for $a(x)$ are pounds per foot, what are the units for da/dx ? What units does $\int_2^8 a(x) dx$ have?