MTH142 Workshop 2: Anti-derivatives

Please work on the following problems with your group.

Warm-Up

1. For each of the following give an example of:
   (a) Two different functions $F(x)$ and $G(x)$ that have the same derivative.
   (b) A function $f(x)$ whose anti-derivative $F(x)$ has a graph which is a line with a negative slope.

2. Write the anti-derivatives to the following functions without the aide of notes or textbook (if you can't today, more studying is required!):

x^n, where $n \neq -1$	$\dfrac{1}{x}$
e^x	a^x, where $a > 0$
$\sin x$	$\cos x$
$\sec^2 x$	$\sec x \tan x$
$\csc^2 x$	$\csc x \cot x$
$\dfrac{1}{1+x^2}$	$\dfrac{1}{\sqrt{1-x^2}}$

Problems

1. Starting with the function for acceleration, show that for motion in a straight line with constant acceleration a, initial velocity v_0, and initial displacement s_0, the displacement after time t is

$$s = \frac{1}{2}at^2 + v_0 t + s_0.$$

2. True/False: For each of the following, choose if each statement is true or false. If true, give an explanation for your answer. If false, give a counterexample.
   (a) If $F(x)$ is an anti derivative of $f(x)$ and $G(x) = F(x) + 2$, then $G(x)$ is an anti derivative of $f(x)$.
   (b) If $F(x)$ and $G(x)$ are both anti derivatives of $f(x)$ on an interval, then $F(x) - G(x)$ is a constant function.
   (c) A rock dropped from a 400-foot cliff takes twice as long to hit the ground as it would if it were dropped from a 200-foot cliff.

3. Consider the following antiderivative problems:

(a) Given that the graph of f passes through the point $(1, -4)$ and that the slope of the tangent line at any point $(x, f(x))$ is $2x + 1$, find the x-intercepts of f.

(b) A particle is moving so that its acceleration at time t is given by $a(t) = \alpha \cos t - \beta \sin t$, where α and β are constants. Its position at time $t = 0$ is $s(0) = 0$, and its velocity at time $t = \pi/2$ is equal to 4. Find the position of the particle at time t. (Your answer may include α and β.)

4. A car is traveling at 30 m/s when the driver sees a roadblock 80 m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid an accident?

5. On the moon the acceleration due to gravity is -5 ft/s^2. An astronaut jumps into the air with an initial upward velocity of 10 ft/s. How high does he go? How long is the astronaut off the ground?

6. Ice forming on a pond at a rate given by

$$\frac{dy}{dt} = k\sqrt{t},$$

where y is the thickness of the ice in inches at time t measured in hours since the ice started forming, and k is a positive constant. Find y as a function of t.