MTH142 Workshop 12: Improper Integrals; Integration Techniques

Warm-Up

(a) For each row, order the numbers from smallest to largest.

$$2^{1/3} \quad 2^{3/4} \quad 2^0 \quad 2^1 \quad 2^2$$

$$10^{1/3} \quad 10^{3/4} \quad 10^0 \quad 10^1 \quad 10^2$$

$$\frac{1}{2^{1/3}} \quad \frac{1}{2^{3/4}} \quad \frac{1}{2^0} \quad \frac{1}{2^1} \quad \frac{1}{2^2}$$

$$\frac{1}{10^{1/3}} \quad \frac{1}{10^{3/4}} \quad \frac{1}{10^0} \quad \frac{1}{10^1} \quad \frac{1}{10^2}$$

(b) Order these integrals from smallest to largest.

$$\int_1^{100} \frac{1}{x^{1/3}}dx \quad \int_1^{100} \frac{1}{x^{1/2}}dx \quad \int_1^{100} \frac{1}{x^0}dx \quad \int_1^{100} \frac{1}{x^1}dx \quad \int_1^{100} \frac{1}{x^2}dx$$

(c) Order these integrals from smallest to largest.

$$\int_{.01}^1 \frac{1}{x^{1/3}}dx \quad \int_{.01}^1 \frac{1}{x^{1/2}}dx \quad \int_{.01}^1 \frac{1}{x^0}dx \quad \int_{.01}^1 \frac{1}{x^1}dx \quad \int_{.01}^1 \frac{1}{x^2}dx$$

Problems

(P1) Which of these integrals are convergent and which are divergent? Try to answer this question based on information from the previous problems, and doing as little computation as possible. (You may have to do some computation).

$$\int_1^{\infty} \frac{1}{x^{1/3}}dx \quad \int_1^{\infty} \frac{1}{x^{1/2}}dx \quad \int_1^{\infty} \frac{1}{x^0}dx \quad \int_1^{\infty} \frac{1}{x^1}dx \quad \int_1^{\infty} \frac{1}{x^2}dx$$

(P2) Which of these integrals are convergent and which are divergent? Try to answer this question based on information from the previous problems, and doing as little computation as possible. (You may have to do some computation).

$$\int_0^1 \frac{1}{x^{1/3}}dx \quad \int_0^1 \frac{1}{x^{1/2}}dx \quad \int_0^1 \frac{1}{x^0}dx \quad \int_0^1 \frac{1}{x^1}dx \quad \int_0^1 \frac{1}{x^2}dx$$

(P3) Suppose that x is a very large number. Order these functions from smallest to largest.

$$e^x \quad e^{-x} \quad e^{x^2} \quad 2^x \quad x^{1/2} \quad x^2 \quad \frac{1}{x^2} \quad \ln x \quad \log_{10} x$$

Suppose that we integrate the above functions from $x = 1$ to $x = \infty$. Which of the functions would give convergent integrals from $x = 1$ to $x = \infty$?

(P4) Integration Techniques Practice: For each problem below, either solve it completely or describe how to solve it. Be specific, for instance, if there's a convenient substitution, what is it?

1. $\displaystyle\int \frac{e^x}{\cos(e^x)}\,dx$
2. $\displaystyle\int y^2(1+\sqrt{y})\,dy$
3. $\displaystyle\int \sqrt{x}\ln x\,dx$
4. $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx$
5. $\displaystyle\int \frac{dt}{t^2+5t+6}$
6. $\displaystyle\int x^2e^x\,dx$
7. $\displaystyle\int \frac{(\sqrt{w}+10)^3}{\sqrt{w}}\,dw$
8. $\displaystyle\int \frac{dx}{e^{2x}+9}$
9. $\displaystyle\int \sqrt{20t-t^2}\,dt$
10. $\displaystyle\int xe^{x^2}\,dx$
11. $\displaystyle\int \sin^{-1}x\,dx$
12. $\displaystyle\int \frac{1+\cos(x)}{\sin(x)}\,dx$
13. $\displaystyle\int \sin^2(z)\cos^3(z)\,dz$
14. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2+4}}$
15. $\displaystyle\int t^3e^{t^2}\,dt$
16. $\displaystyle\int \frac{dk}{k^2-6k+9}$
17. $\displaystyle\int e^{2x}\ln(e^x)\,dx$
18. $\displaystyle\int \sin(2x)\cos(x)\,dx$
19. $\displaystyle\int x\ln(x)\,dx$
20. $\displaystyle\int \frac{e^u}{\sqrt{16-e^{2u}}}\,du$
21. $\displaystyle\int \frac{e^{2t}}{e^t-2}\,dt$
22. $\displaystyle\int x\tan^{-1}x\,dx$
23. $\displaystyle\int \frac{2x^2-x+4}{x^3+4x}\,dx$
24. $\displaystyle\int x^3\sin(x^2)\,dx$
25. $\displaystyle\int x^5\sec^5(3x^6)\tan^3(3x^6)\,dx$
26. $\displaystyle\int \frac{dx}{\sqrt{6-x^2+10x}}$
27. $\displaystyle\int \frac{3y^5+28y}{y^4+9}\,dy$
28. $\displaystyle\int \frac{4x^2+3x+6}{x^2(x^2+3)}\,dx$
29. $\displaystyle\int_0^{9/2} \frac{x^3}{\sqrt{81-x^2}}\,dx$