MTH142 Workshop 11: Method of Partial Fractions

Warm-Up

1. Write out the partial fraction decomposition of the function, but **do not evaluate the values of the coefficients**.

(a) $\frac{x}{x^2+x-2}$ (c) $\frac{10}{5x^2-2x^3}$ (e) $\frac{1}{x^3-1}$

(b) $\frac{x^3-2x^2-4}{x^3-2x^2}$ (d) $\frac{x^4}{(x^2+2x-1)(x^2+2)^2}$ (f) $\frac{x^6}{x^2-4}$

2. What is **wrong** with the following partial fraction decomposition, and what should the correct decomposition be?

$$\frac{1}{(x^2-9)^2} = \frac{Ax+B}{(x^2-9)^2} + \frac{Cx+D}{x^2-9}$$

3. Solve the following integral by taking a u-substitution.

$$\int \frac{x+1}{x^2+2x+5}dx$$

Problems

4. Find the partial fraction decompositions of the following and evaluate the coefficients.

(a) $\frac{x}{x^2+x-2}$ (b) $\frac{10}{5x^2-2x^3}$ (c) $\frac{1}{x^3-1}$

5. Solve the following integrals.

(a) $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}dx$

(b) $\int \frac{10}{(x-1)(x^2+9)}dx$

(c) $\int \frac{\sqrt{x+4}}{x}dx$

Hint: Take $u = \sqrt{x+4}$.

6. Consider the following integral.

$$\int \frac{x+4}{x^2+2x+5}dx$$

   (a) We would like to do a u-substitution for the denominator, as in Warm-up 3. However, we cannot with the numerator as is. To fix this, split the fraction into the sum of two fractions, where the first is the integrand from Warm-Up 3.

   (b) Split the integral into the sum of two integrals and solve. We know the answer for the first integral, but it remains to solve the second.
   Hint: Complete the square.

7. Evaluate the following integral. You will need techniques and answers from previous problems.

$$\int \frac{1}{x^3-1}dx$$