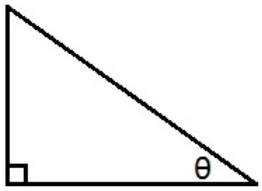
MTH142 Workshop 10: Trigometric Substitution

1. Label the triangle for each of the following substitutions. Then make the substitution in the given function to write in terms of θ and use a Pythagorean identity to simplify.



(a) $x = a\sin\theta, \quad f(x) = a^2 - x^2$

(b) $x = a\tan\theta, \quad g(x) = a^2 + x^2$

(c) $x = a\sec\theta, \quad h(x) = x^2 - a^2$

2. Evaluate the following integrals. It may help to recall the trig identity:

$$\sin(2\theta) = 2\cos\theta\sin\theta$$

(a) $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}dx$

(b) $\displaystyle\int \sqrt{1-4x^2}dx$

3. Each integral below can be done using a trig substitution or a u-substitution. Try it both ways and compare your answers.

(a) $\displaystyle\int \frac{x}{\sqrt{x^2+16}}dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+16}}dx$

4. Try using a u-substitution on the integral below similar to the last problem, and describe why it will not work. Then use a trig-substitution to evaluate the integral.

$$\int \frac{x^2}{\sqrt{16-x^2}}dx$$

5. In each part below, solve the following integral by first completing the square, then by taking a relevant trig substitution:

(a) $\displaystyle\int \frac{dz}{\sqrt{z^2+2z+5}}$

(b) $\displaystyle\int \frac{dz}{(z^2+2z+5)^{3/2}}$

(c) $\displaystyle\int \frac{dz}{(z^2+2z+5)^2}$