MTH142 Workshop 1: Curve Sketching; Optimization

1. What is the area of the largest rectangle that can be inscribed in a circle of radius r? (Your answer will be in terms of r.)

2. A pizza parlor has the following special: For \$2, you can order a slice of pizza as large as you want, as long as the perimeter of the slice is no more than 24 inches. (A slice of pizza is in the shape of a sector of a circle.) What diameter should the pizza have in order to give you the largest slice? (Here "largest" means "largest area.")

3. Hank has a piece of cardboard that is 5 in. by 8 in. He needs to construct an open-top box by cutting a square from each corner and folding up the sides. (**Note:** The square cut from each corner is the same size for all four corners.) What is the largest possible volume of such a box?

4. Sketch a graph of a function $f(x)$ that satisfies all of the given conditions:

(a) $f(-2) = f(2) = f(6) = 0$.

(b) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = 3$.

(c) $\lim_{x\to1} f(x) = \infty$.

(d) $f'(4) = 0$.

(e) $f'(x) > 0$ if $x < 1$ or $x > 4$, and $f'(x) < 0$ if $1 < x < 4$.

(f) $f''(x) > 0$ if $x < 1$ or $1 < x < 7$, and $f''(x) < 0$ if $x > 7$.

5. Let $f(x) = \dfrac{\ln x}{x}$. Note that

$$f'(x) = \frac{1 - \ln x}{x^2} \text{ and } f''(x) = \frac{2\ln x - 3}{x^3}.$$

Complete the following with your group, **without the aid of a calculator**:

(a) Find the domain of f.
(b) Find all x- and y-intercepts.
(c) Find all vertical asymptotes.
(d) Find the limits at $\pm\infty$ and any horizontal asymptotes.
(e) Where is $f(x)$ increasing? decreasing? Where does f have a local maximum/minimum?
(f) Where is $f(x)$ concave up? concave down? Where does f have an inflection point?
(g) Use all of this information to sketch a graph of f.

6. Let $f(x) = xe^{-x^2}$. Complete the same steps as the previous problem, **without the aid of a calculator.**