Math 142: Calculus II

Midterm 2

April 5, 2018

NAME (please print legibly): Solutions

Your University ID Number: ______________________

Indicate the lecture time you are registered for with a check in the appropriate box:

Gafni	TR 9:40-11:55pm	
Gafni	TR 2:00-3:15pm	
Passant	TR 3:25-4:40pm	
Zeng	MW 9:00-10:15am	

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 11 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- **Show all work and justify all answers.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Numerical or algebraic simplifications of answers are not required, **except when specifically stated otherwise.**
- Please sign the pledge below.

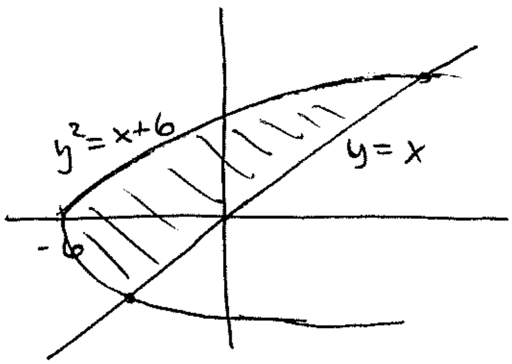
Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________

QUESTION	VALUE	SCORE
1	18	
2	18	
3	12	
4	18	
5	12	
6	22	
TOTAL	100	

1. (18 points) Find the area enclosed by the line $y = x$ and the parabola $y^2 = x + 6$.



Write as functions of y

$x = y \qquad x = y^2 - 6$

Intersection points:

$$y^2 - 6 = y$$
$$y^2 - y - 6 = 0$$
$$(y-3)(y+2) = 0$$
$$y = -2, 3$$

$$\text{Area} = \int_{-3}^{2} y - (y^2 - 6)\,dy$$

$$= \frac{1}{2}y^2 - \frac{1}{3}y^3 + 6y \Big|_{-2}^{3}$$

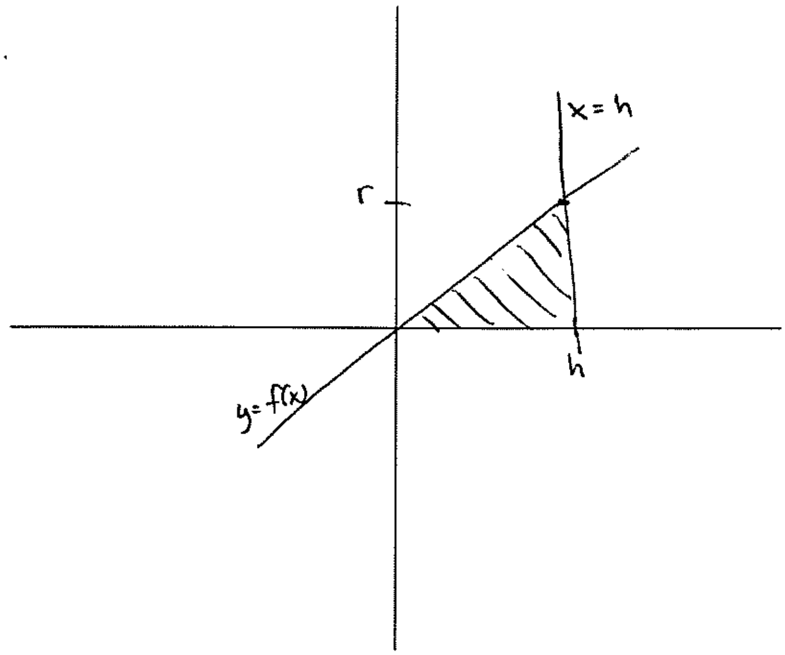
$$= \left(\frac{9}{2} - 9 + 18\right) - \left(2 + \frac{8}{3} - 12\right)$$

$$= \boxed{19 + \frac{9}{2} - \frac{8}{3}}$$

2. (18 points) Let h and r be some arbitrary constants. Consider the curve

$$f(x) = \left(\frac{r}{h}\right) x.$$

(a) On the axis below, sketch and shade the region enclosed by the curves $y = f(x)$, $y = 0$ and $x = h$.



(b) What is the name of the solid formed when we rotate the region enclosed by the curves $y = 0$, $x = h$ and $y = f(x)$ around the x-axis?
[Give the name of the specific shape, e.g. "Cube", not just "solid of revolution"]

Cone

(c) Using the method of discs/washers, find the volume of rotation of the region enclosed by the curves $y = 0$, $x = h$ and $y = f(x)$ when rotated around the x-axis.

Only one function $\Rightarrow$ discs

$$\text{Volume} = \int_0^h \pi (\text{radius})^2 \, dx \qquad \text{radius} = f(x) = \left(\frac{r}{h}\right)x$$

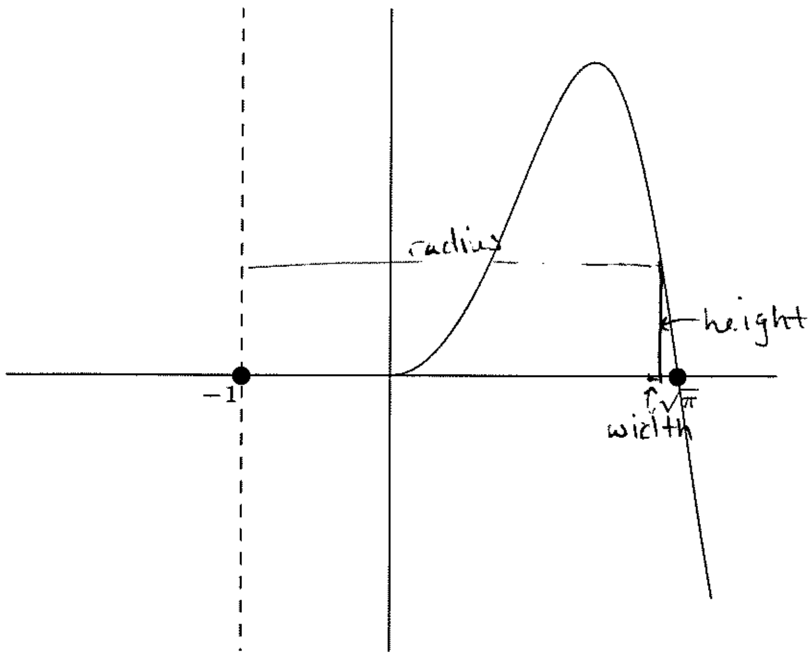
$$V = \int_0^h \pi \left[\left(\frac{r}{h}\right)x\right]^2 dx.$$

$$= \frac{\pi r^2}{h^2} \int_0^h x^2 \, dx$$

$$= \frac{\pi r^2}{h^2} \left[\frac{1}{3}x^3\right]_0^h$$

$$= \frac{\pi r^2}{h^2}\left(\frac{1}{3}h^3 - 0\right) = \boxed{\frac{1}{3}\pi r^2 h}$$

3. (12 points) Consider the volume obtained by rotating the region under the curve $y = \sin(x^2)$ between $x = 0$ and $x = \sqrt{\pi}$ around the line $x = -1$. Set up an integral equal to the volume of this solid. **DO NOT SOLVE THIS INTEGRAL.**

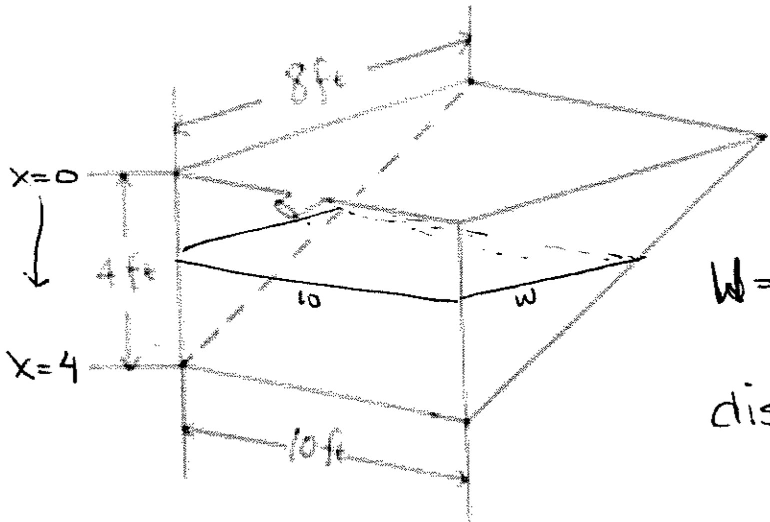


$$V = \int_0^{\sqrt{\pi}} 2\pi (\text{radius})(\text{height})\, dx$$

$$= \int_0^{\sqrt{\pi}} 2\pi \left(x - (-1)\right) f(x)\, dx.$$ $f(x) = \sin(x^2).$

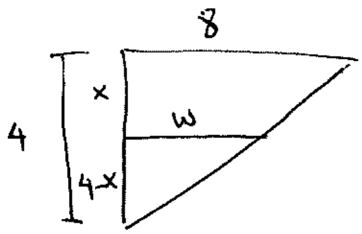
$$\boxed{V = \int_0^{\sqrt{\pi}} 2\pi (x+1) \sin(x^2)\, dx}$$

4. (18 points) A tank (see the Figure below) is full of a liquid that weighs 75 lb/ft^3. Set up an integral that can be used to compute the work required to pump the liquid out of the spout. **DO NOT SOLVE THIS INTEGRAL.**



$$W = \int_0^4 (\text{distance})(\text{weight})$$

$$\text{distance} = x$$



$$\text{Volume (of cross-section)} = 10 \cdot w \cdot dx.$$
$$= 10(2)(4-x)dx$$

$$\text{Weight of cross-section} = \text{Vol.} \cdot 75$$
$$= 75(20)(4-x)dx.$$

$$\frac{8}{4} = \frac{w}{4-x}$$

$$w = 2(4-x)$$

$$\boxed{\text{Work} = \int_0^4 x \cdot 75 \cdot 20 \cdot (4-x)\, dx}$$

5. (12 points)

(a) Find the average value of the function $f(x) = 2\sqrt{x}$ on the interval $[0, 4]$.

$$f_{ave} = \frac{1}{4-0}\int_0^4 2\sqrt{x}\,dx$$

$$= \frac{1}{4}\left[2\left(\frac{2}{3}\right)x^{3/2}\right]_0^4$$

$$= \frac{1}{3}\left(4^{3/2} - 0\right) = \boxed{\frac{8}{3}}$$

(b) Find the point(s) in the interval $(0, 4)$ at which $f(x)$ is equal to its average value.

$$f(c) = f_{ave}$$

$$2\sqrt{c} = \frac{8}{3}$$

$$\sqrt{c} = \frac{4}{3}$$

$$\boxed{c = \frac{16}{9}}$$

$$\frac{9}{9} < \frac{16}{9} < \frac{18}{9}$$

so $1 < \frac{16}{9} < 2 \Rightarrow \frac{16}{9}$ is in $(0,4)$.

6. (22 points) Evaluate the following integrals.

Int. by parts:
$\int u\,dv = uv - \int v\,du$

(a) $\int xe^x \, dx$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$\int xe^x dx = xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$

(b) $\int e^{2x} \sin x \, dx$

$u = e^{2x} \qquad dv = \sin x\, dx$

$du = 2e^{2x} dx \qquad v = -\cos x$

$\int e^{2x} \sin x\, dx = -e^{2x}\cos x - \int -2e^{2x} \cos x\, dx$

$= -e^{2x}\cos x + 2\int e^{2x} \cos x\, dx.$

$\hookrightarrow u = e^{2x} \qquad dv = \cos x\, dx$

$du = 2e^{2x} dx \qquad v = \sin x$

$\int e^{2x} \sin x\, dx = -e^{2x}\cos x + 2\left(e^{2x}\sin x - \int 2e^{2x} \sin x\, dx\right)$

$\int e^{2x} \sin x\, dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x} \sin x\, dx.$

$5\int e^{2x} \sin x\, dx = -e^{2x}\cos x + 2e^{2x}\sin x$

$\boxed{\int e^{2x} \sin x\, dx = \frac{1}{5}\left(-e^{2x}\cos x + 2e^{2x}\sin x\right) + C}$

Evaluate the following integral.

(c) $\int (\ln x)^2\, dx$

$u = (\ln x)^2 \qquad dv = dx$

$du = \frac{2\ln x}{x} dx \qquad v = x$

$\int (\ln x)^2 dx = x(\ln x)^2 - \int 2\ln x\, dx$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x} dx \quad v = x.$

$\int (\ln x)^2 dx = x(\ln x)^2 - 2\left(x\ln x - \int dx\right)$

$= \boxed{x(\ln x)^2 - 2x\ln x + 2x + C}$

Blank page for scratch work