

# Math 142: Calculus II

Midterm 2

November 16, 2017

NAME (please print legibly): \_\_\_\_\_

Your University ID Number: \_\_\_\_\_

Indicate your instructor with a check in the appropriate box:

Crossen	MW 9-10:15	<input type="checkbox"/>
Zhong	MW 3:25-4:40	<input type="checkbox"/>

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 9 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.
- Please sign the pledge below.

## Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: \_\_\_\_\_

QUESTION	VALUE	SCORE
1	15	
2	15	
3	10	
4	10	
5	10	
6	15	
TOTAL	75	

1. (15 points) Evaluate the following indefinite integrals.

(a)  $\int x^3 \ln x \, dx.$

(b)  $\int \sec^4 x \, dx.$

**2. (15 points)** Evaluate the following definite integrals. Your answer should **NOT** involve any trigonometric functions.

(a)  $\int_0^1 \frac{x^2}{1+x^6} dx.$

(b)  $\int_0^{\pi/6} \sin^3 x dx.$

**3. (10 points)** Consider the function  $f(x) = 3x^2 - 12x - 10$ .

(a) For  $b > 0$ , compute the average value of  $f(x)$  on the interval  $0 \leq x \leq b$ . **Note:** Your answer should be a function of  $b$ .

(b) Find all numbers  $b > 0$  such that the average value of  $f(x)$  on  $[0, b]$  is equal to 6.

**4. (10 points)** The following problems concern the solid of revolution generated by rotating about a given axis the region  $R$ , which is enclosed by the curve  $y = x^2$  and the curve  $y = 2x$ . You may use either the method of disks/washers or the method of cylindrical shells, but you must clearly indicate which one you are using in each problem.

(a) If  $R$  is rotated about the  $x$ -axis, set up but do not evaluate an integral for computing the volume of the resulting solid.

(b) If  $R$  is rotated about the  $y$ -axis, set up but do not evaluate an integral for computing the volume of the resulting solid.

**5. (10 points)** Consider the functions  $f(x) = \sin x$  and  $g(x) = \cos x$ . Compute the area between the graphs of these two functions on the interval  $[-\pi/2, \pi/2]$ . Your answer should **NOT** involve any trigonometric functions.

**6. (15 points)** A tank has the shape of an inverted circular cone with height 6m and base radius 3m. It is filled with water to a height of 4m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is  $1000 \text{ kg/m}^3$ .)

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**Formula Sheet**

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$
- $\sin(2x) = 2 \sin x \cos x$
- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\sin x \cos y = \frac{\sin(x - y) + \sin(x + y)}{2}$
- $\sin x \sin y = \frac{\cos(x - y) - \cos(x + y)}{2}$
- $\cos x \cos y = \frac{\cos(x - y) + \cos(x + y)}{2}$