MATH 142

Final

May 5, 2014

NAME (please print legibly): _____

Your University ID Number: _____

Circle your Instructor's Name along with the Lecture Time:

Yoonbok Lee (MWF 9:00)	Dillon Ethier (MWF 12:00)
Carl Mueller (MWF 1:00)	Eyvindur Palsson (TR 2:00)

- **No calculators are allowed on this exam.**
- **Your score on part A can make up for a bad midterm. But part A counts towards your score on the final, so please do not skip part A, even if you're satisfied with your midterm scores.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your simplified final answers in the spaces provided.**

Part A		
QUESTION	VALUE	SCORE
1	24	
2	15	
3	19	
4	12	
5	15	
6	15	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
1	19	
2	22	
3	13	
4	13	
5	22	
6	11	
TOTAL	100	

Part A

1. (24 points)

(a), (6 points) Find the vertical and horizontal asymptotes of

$$f(x) = \frac{3x^2 + 2x + 4}{2x^2 + x - 1}.$$

ANSWER: _____

(b), (6 points) Does the following function have any symmetry? If so, what kind of symmetry does it have?

$$f(x) = \frac{x^3 \sin(x)}{2 + x^2} - e^{x^2}$$

ANSWER: _____

(c), (6 points) Find the intervals of increase and decrease for the following function. Then find the points x where the function has a local maximum or local minimum.

$$f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 2x + 5$$

ANSWER: _____

(d), (6 points) Using the same function $f(x)$ as in part (c), find the intervals on which the function is concave up and concave down, and find the points of inflection.

ANSWER: _____

2. (15 points) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 9 - x^2$ above the x-axis. What are the dimensions of such a rectangle with the greatest possible area?

ANSWER: _____

3. (19 points)

Evaluate the following integrals.

(a) (6 points)

$$\int x \sec^2(x^2+1)dx$$

ANSWER: _____

(b) (7 points)

$$\int (x+1)^{555}x^2dx$$

ANSWER: _____

(c) (6 points)

$$\int_0^{\pi} \sin(x) e^{\cos(x)} dx$$

ANSWER: _____

4. (12 points) Find the area enclosed by the curves $y = 6x^2$ and $y = 2x^3$

ANSWER: _____

5. (15 points) Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^3$, $y = x$ about the line $y = 4$ using the method of discs or washers.

ANSWER: _____

6. (15 points) Find the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$, $y = 0$ about the line $x = 2$ using cylindrical shells.

ANSWER: _____

Part B

1. (19 points) The great pyramid of Giza is approximately 140 meters high, and the base is a square of side length approximately 440 meters. Suppose that Egypt's ruling generals decide to hollow out the pyramid and use it as a water tower. Compute the work required to pump water from the base of the pyramid until it completely fills the space available.

Since you are not allowed to have a calculator, you do not have to completely simplify your answer.

Hint: Let x be the distance from the top of the pyramid. A horizontal slice of the pyramid at position x is a square, and by using some geometry you can compute the side length of this square as a function of x.

ANSWER: _____

2. (22 points) (a) (11 points) Evaluate

$$\int_0^{\pi} x \sin(2x) dx$$

ANSWER: _____

(b) (11 points) Evaluate

$$\int x^2 e^{3x} dx$$

ANSWER: _____

3. (13 points) Evaluate the integral

$$\int \sec^4(\theta) d\theta.$$

ANSWER: _____

4. (13 points) Evaluate the integral

$$\int_0^{\sqrt{3}/2} \frac{x^3}{\sqrt{1-x^2}} dx.$$

ANSWER: _____

5. (22 points)

Evaluate the following integrals.

(a) (11 points)

$$\int \frac{1}{2x^2 - 7x - 15} dx$$

ANSWER: _____

(b) (11 points)

$$\int \frac{x^2 + 2x + 2}{x - 2} dx$$

ANSWER: _____

6. (11 points) Find the arc length of the curve $y = 2(x-1)^{\frac{3}{2}}$ for $1 \leq x \leq 3$.

ANSWER: _____