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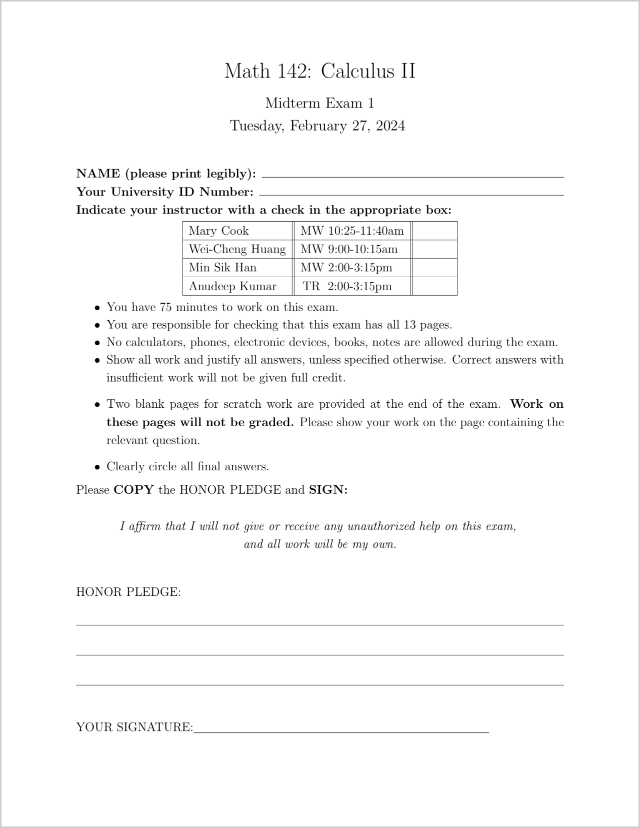

Math 142: Calculus II

Midterm Exam 1
Tuesday, February 27, 2024

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Indicate your instructor with a check in the appropriate box:

Mary Cook	MW 10:25-11:40am	☐
Wei-Cheng Huang	MW 9:00-10:15am	☐
Min Sik Han	MW 2:00-3:15pm	☐
Anudeep Kumar	TR 2:00-3:15pm	☐

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 13 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers, unless specified otherwise. Correct answers with insufficient work will not be given full credit.
- Two blank pages for scratch work are provided at the end of the exam. **Work on these pages will not be graded.** Please show your work on the page containing the relevant question.
- Clearly circle all final answers.

Please **COPY** the HONOR PLEDGE and **SIGN:**

I affirm that I will not give or receive any unauthorized help on this exam,
and all work will be my own.

HONOR PLEDGE:

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YOUR SIGNATURE: ____________________

FOR REFERENCE, NO QUESTION ON THIS PAGE

Table of common antiderivatives:

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x)+g(x)$	$F(x)+G(x)$	$\sec^2 x$	$\tan x$
$x^n (n \neq 1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$\frac{1}{x}$	$\ln \lvert x \rvert$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\frac{1}{1+x^2}$	$\tan^{-1} x$

Fundamental Theorem of Calculus:

(i) Suppose f is continuous on $[a,b]$ and differentiable on (a,b). If $g(x) = \int_a^x f(t)\,dt$, then

$$g'(x) = f(x)$$

for all $x \in (a,b)$.

(ii) Suppose F is an antiderivative of f, and f is continuous on $[a,b]$ and differentiable on (a,b). Then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

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1. (15 points)

Consider the curve $y = x^3 + 3x^2$. $= f(x)$

(a) Find the x- and y-intercepts of the curve.

x-int: $x^3+3x^2=0 \Rightarrow x^2(x+3)=0 \Rightarrow x=0$ or -3

$(0,0), (-3,0)$

y-int.: $(0,0)$.

(b) Find any asymptotes of the curve.

None

(c) Find the intervals of increase and decrease for the curve.

$f'(x) = 3x^2+6x$
$= 3x(x+2)$

$f' = 0 \Rightarrow x = 0$ or -2
f' DNE $\Rightarrow$ No sol.

$$f' \quad \begin{array}{ccccc} + & & - & & + \\ & -2 & & 0 & \end{array} \rightarrow x$$

$f''(x) = 6x+6$

$f''=0 \Rightarrow x=-1$
f'' DNE $\Rightarrow$ No sol.

$$f'' \quad \begin{array}{ccc} - & & + \\ & -1 & \end{array} \rightarrow$$

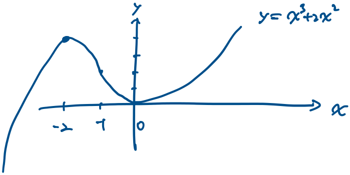
Increasing on:
$(-\infty,-2)$, $(0,+\infty)$

Decreasing on:
$(-2,0)$

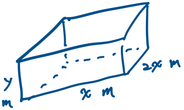
Concave up on: $(-1,\infty)$
Concave down on: $(-\infty,-1)$

(d) Sketch the curve.

x		-2		-1		0	
f'	$+$		$-$		$-$		$+$
f''	$-$		$-$		$+$		$+$
f	↗	4	↘	2	↘	0	↗



2. (10 points) A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs 9 dollars per square meter. Material for the sides costs 6 dollars per square meter. Find the cost of the materials for the cheapest such container.



C : cost, in \$.

$2x^2y = 10 \Rightarrow y = \frac{5}{x^2}$

$C = 2x^2 \cdot 9 + 6(2xy + 4xy)$

$= 18x^2 + 36xy$

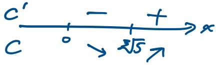
$\Rightarrow C(x) = 18x^2 + 180\frac{1}{x}$

Minimize $C(x)$ on $x>0$.

$C'(x) = 36x - 180\frac{1}{x^2} = \frac{36x^3 - 180}{x^2}$

$C' = 0 \Rightarrow 36x^3 = 180 \Rightarrow x^3 = 5 \Rightarrow x = \sqrt[3]{5}$

C' DNE $\Rightarrow x = 0$



The min cost $= C(\sqrt[3]{5}) = 18\sqrt[3]{25} + 180\frac{1}{\sqrt[3]{5}}$ dollars

3. (10 points)

(a) Consider the function

$$g(x) = \int_x^2 \sin^2(t)\, dt.$$

Compute $g'(x)$.

$$g'(x) = -\sin^2 x$$

(b) Consider the function

$$h(x) = \int_0^{x^3} 5t^4\, dt.$$

Compute $h'(x)$.

$$h'(x) = 5\,(x^3)^4 \cdot 3x^2$$

4. (15 points) Water is poured into a 2000cm^3 bucket at a rate of $r(t) = 10t$ cm^3/second. At time $t = 0$ seconds, the bucket is empty.

(a) How much water is in the bucket after 10 seconds?

$$\int_0^{10} 10t\, dt = 10 \cdot \frac{1}{2} t^2 \Big|_0^{10}$$

$$= 500 \text{ cm}^3$$

(b) Find an expression for the amount of water in the bucket after x seconds, for x between

0 and the time the bucket fills.

$$\int_0^x 10t\,dt$$

(c) How long will it take for the bucket to fill?

$$2000 = \int_0^x 10t\,dt = 5x^2$$

$$\Rightarrow x^2 = 400 \Rightarrow x = 20\ s$$

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5. (10 points) Evaluate the following indefinite integrals.

(a) $\int 3x^5(1-x^3)^{\frac{1}{4}}\ dx$.

$$\overset{u=1-x^3}{\underset{du=-3x^2\,dx}{=}} -\int(1-u)\,u^{\frac{1}{4}}\,du$$

$x^3 = 1-u$

$$= -\int u^{\frac{1}{4}} - u^{\frac{5}{4}}\,du = -\left(\frac{4}{5}u^{\frac{5}{4}} - \frac{4}{9}u^{\frac{9}{4}}\right) + C$$

$$= -\frac{4}{5}(1-x^3)^{\frac{5}{4}} + \frac{4}{9}(1-x^3)^{\frac{9}{4}} + C$$

(b) $\int \cos\theta \cdot e^{\sin\theta}\ d\theta$.

$$\overset{u=\sin\theta}{\underset{du=\cos\theta d\theta}{=}} \qquad \int e^u\,du = e^u + C$$

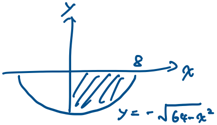
$$= e^{\sin\theta} + C$$

6. (10 points) Evaluate the integral **by interpreting it in terms of areas**.

(*Hint*: The area of a circle of radius r is πr^2. The area of a triangle with base b and height h is $\frac{1}{2}bh$. The area of a rectangle with length l and width w is lw.)

(a) $\int_0^8 -\sqrt{64-x^2}\,dx$

$$y = -\sqrt{64-x^2} \Rightarrow x^2+y^2=8^2$$

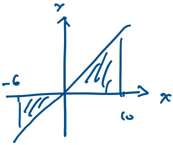


$$\int_0^8 -\sqrt{64-x^2}\,dx$$

$$= -\frac{1}{4}\pi\cdot 8^2$$

$$= -16\pi$$

(b) $\int_{-6}^{10} x\,dx$



$$\int_{-6}^{10} x\,dx = \frac{1}{2}\cdot 10^2 - \frac{1}{2}\cdot 6^2$$

$$= 50 - 18 = 32$$

7. (15 points)
Consider a particle moving along a line. At $t = 0$, the particle is located at position 0, moving with velocity 6 (m/s). Suppose that the acceleration of the particle is given by

$$a(t) = -4t - 4 \text{ (m/s}^2\text{)}.$$

(a) When does the particle stop (i.e., its velocity becomes zero) for the first time after $t = 0$?

$$v(t) = \int a(t)\,dt = -2t^2 - 4t + C$$

$$v(0) = 6 \Rightarrow C = 6$$

$$v(t) = 0 \Rightarrow -2t^2 - 4t + 6 = 0$$

$$\Rightarrow t^2 + 2t - 3 = 0 \Rightarrow (t+2)(t-1) = 0$$

$$\Rightarrow t = 1 \text{ (s)}$$

(b) Compute the **total distance traveled** by the particle between $t = 0$ and $t = 2$.

$$\int_0^2 |v(t)|\,dt$$

$$= \int_0^1 -2t^2 - 4t + 6\,dt - \int_1^2 -2t^2 - 4t + 6\,dt$$

$$= \left(-\frac{2}{3}t^3 - 2t^2 + 6t\right)\Big|_0^1 - \left(-\frac{2}{3}t^3 - 2t^2 + 6t\right)\Big|_1^2$$

$$= -\frac{2}{3} - 2 + 6 - \left(-\frac{2}{3}\cdot 7 - \cancel{2\cdot 3} + \cancel{6}\right)$$

$$= 8 \text{ m}$$

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8. (15 points)
(a) Find the function $f(x)$ and a number b, such that the area under the graph of $y = f(x)$ from $x = 0$ to $x = b$ is estimated by the following Riemann sum.

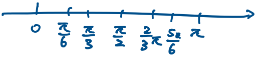
$$\sum_{i=0}^{14} \frac{1}{3}\left(\frac{i^2}{9} + 4\right)$$

$$\Delta x = \frac{1}{3} = \frac{b-0}{15} \Rightarrow b = 5$$

$$f(x) = x^2 + 4$$

(b) Estimate the area under the graph of $y = (\sin x)^2$ from $x = 0$ to $x = \pi$ using the Riemann sum with 6 intervals. Use left endpoints as the sample points. Your answer should be a number, but you don't need to fully simplify.

$$\text{Area} = \int_0^\pi (\sin x)^2 \, dx$$



$$\Delta x = \frac{\pi}{6}$$

$$L_6 = \frac{\pi}{6}\Big[(\sin 0)^2 + \left(\sin \frac{\pi}{6}\right)^2 + \left(\sin \frac{\pi}{3}\right)^2 + \left(\sin \frac{\pi}{2}\right)^2 + \left(\sin \frac{2\pi}{3}\right)^2 + \left(\sin \frac{5\pi}{6}\right)^2\Big]$$

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