Math 142: Calculus II

Midterm 1

March 1, 2018

NAME (please print legibly): Solutions
Your University ID Number: ______________________
Indicate the lecture time you are registered for with a check in the appropriate box:

Gafni	TR 9:40-10:55pm	
Gafni	TR 2:00-3:15pm	
Passant	TR 3:25-4:40pm	
Zeng	MW 09:00-10:15am	

- You have 75 minutes to work on this exam.
- You are responsible for checking that this exam has all 12 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers. $\boxed{\text{Box}}$ final answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Numerical or algebraic simplifications of answers are not required, **except when specifically stated otherwise.**
- Please sign the pledge below.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________

QUESTION	VALUE	SCORE
1	19	
2	16	
3	15	
4	10	
5	10	
6	15	
7	15	
TOTAL	100	

1. (19 points)

Consider the function $f(p) = \dfrac{1}{p^2-9}$ and its derivatives $f'(p) = \dfrac{-2p}{(p^2-9)^2}$, $f''(p) = \dfrac{6p^2+18}{(p^2-9)^3}$.

(a) Express the domain of $f(p)$ in interval notation.

$p^2 - 9 \neq 0$

$p^2 \neq 9$ Domain $= (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$p \neq \pm 3$

(b) Find all values of p where vertical asymptotes exist. If none exist note this.

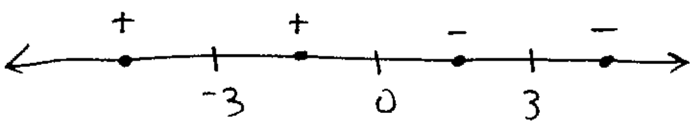
$f(p)$ has vertical asymptotes for $p = \pm 3$

since $f(\pm 3) = \frac{1}{0}$.

(c) Find the intervals where $f(p)$ is increasing. Give your answer in interval notation, if f is never increasing, then state this.

$f(p)$ is increasing when $f'(p) > 0$, so we check for critical points and use a number line:

$f'(p) = 0$ when $p = 0$ and $f'(p)$ is not defined when $p = \pm 3$



$f(p)$ is increasing on $(-\infty, -3) \cup (-3, 0)$

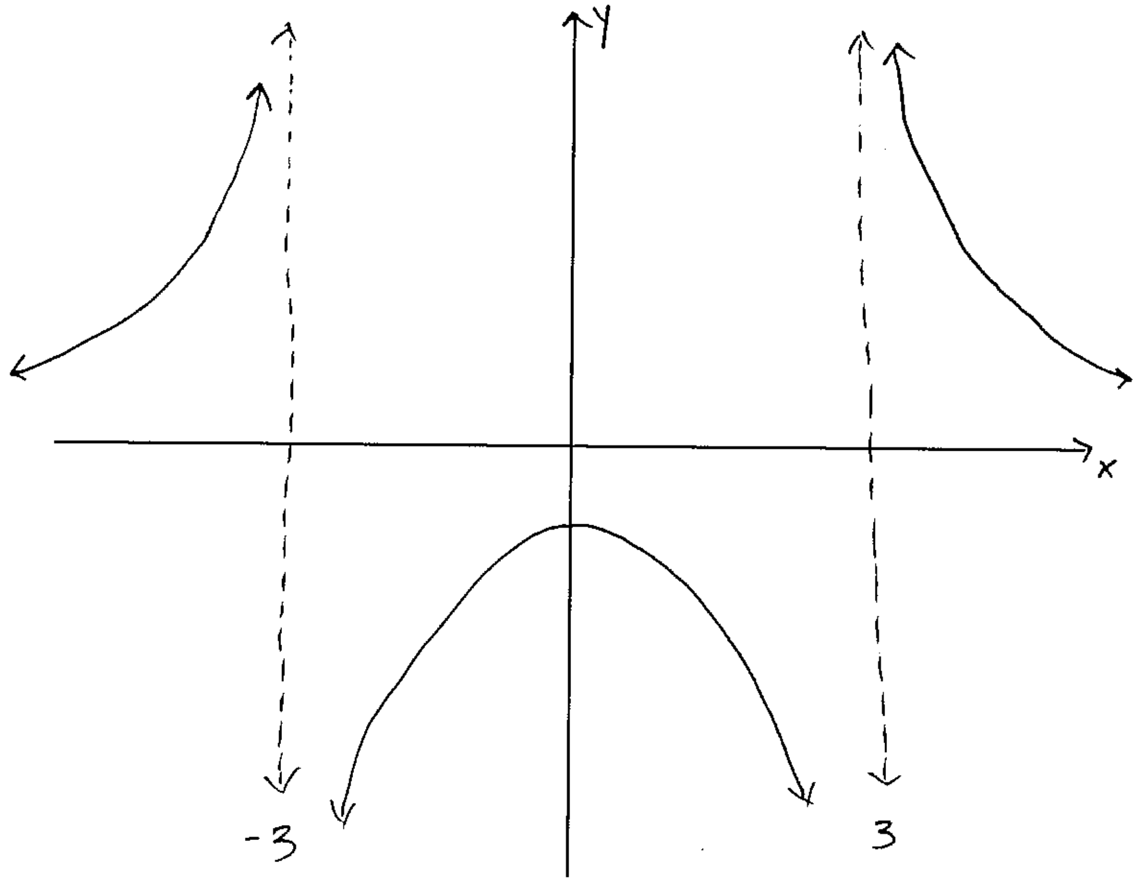
(d) Find any horizontal asymptotes of $f(p)$, if none exist note this.

$$\lim_{p \to \infty} \frac{1}{p^2-9} = 0$$

$$\lim_{p \to -\infty} \frac{1}{p^2-9} = 0$$

$f(p)$ has a horizontal asymptote at $y=0$

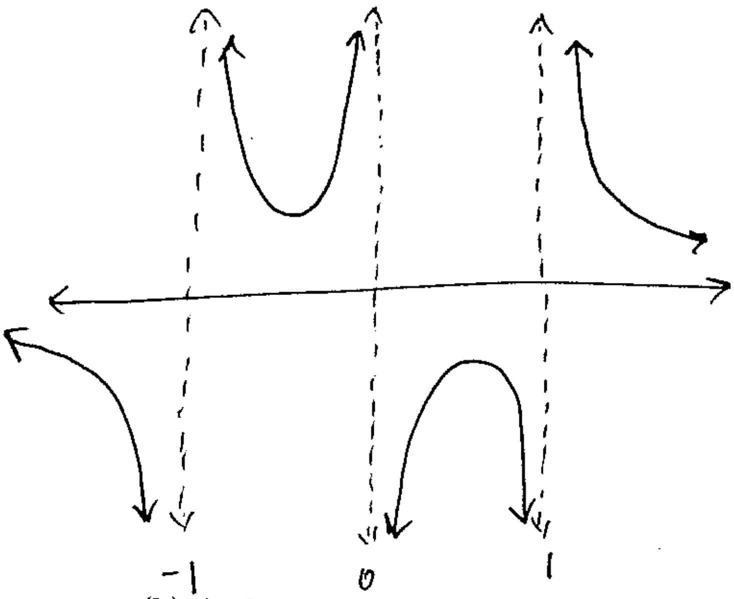
(e) Use the above information to sketch the graph of $f(p)$ on the axis below.



2. (16 points) Indicate whether the following statements TRUE or FALSE. If the statement is FALSE please give a brief explanation of why. If the statement is TRUE please sketch a graph of a function (on your own set of axes) which has the desired property.

(a) A function can have three different vertical asymptotes.

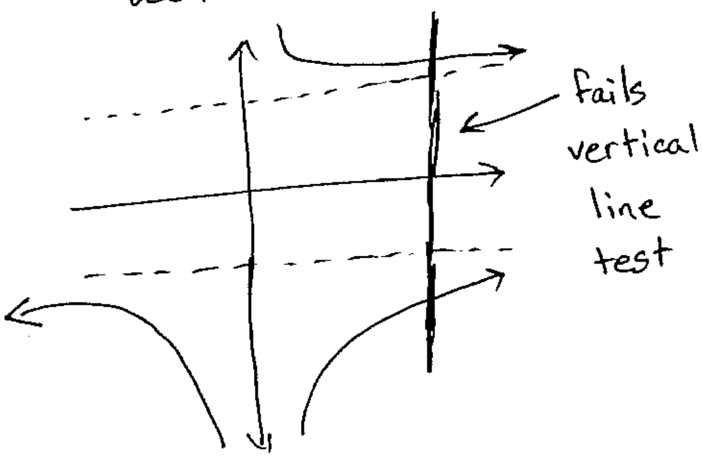
TRUE Example : $f(x) = \frac{1}{(x-1)x(x+1)}$



(b) A function can have three different horizontal asymptotes.

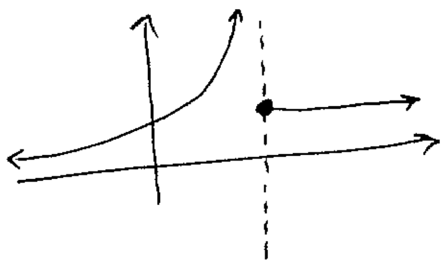
FALSE

The function would fail the vertical line test :



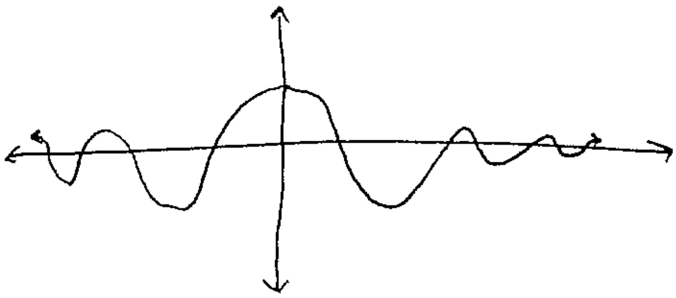
(c) A function can cross its own vertical asymptote.

FALSE
one side has to go to infinity; to avoid failing the vertical line test, a function can, at most, be defined at an asymptote; not cross :



(d) A function can cross its own horizontal asymptote.

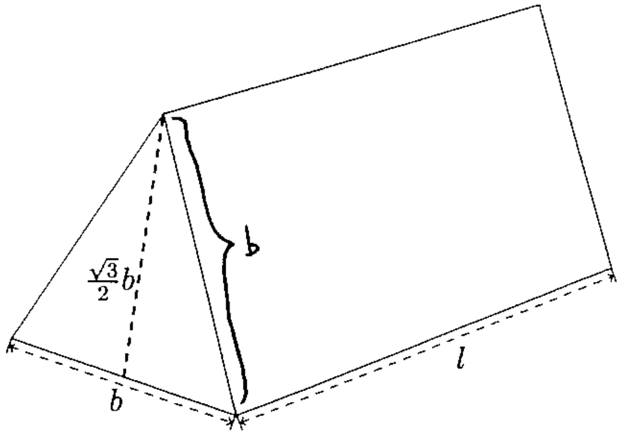
TRUE Example : $f(x) = \frac{\sin(x)}{x}$



$\lim_{x \to \infty} \frac{\sin(x)}{x} = 0$ by the squeeze theorem.

3. (15 points) A Swiss sweet company has developed a new type of mint. After a meeting between the research department and the people from marketing, the shape decided for the new mint was triangular prism with ends forming equilateral triangles. Each mint has a volume of $\frac{27}{4}$cm^3. The company wishes to minimize the packaging needed to cover each mint.

Find the length b of the base of the triangle that will minimize the packaging needed to cover the mints.



Volume = (Area of Base) × (Length)

$= \left(\frac{1}{2} b h\right) \times \ell$

$= \left(\frac{1}{2} \cdot b \cdot \frac{\sqrt{3}}{2} b\right) \cdot \ell$

$= \frac{\sqrt{3}}{4} b^2 \ell$

$\frac{\sqrt{3}}{4} b^2 \ell = \frac{27}{4} \Rightarrow \ell = \frac{27}{\sqrt{3}\, b^2}$

$SA(b,\ell) = 2\left(\frac{1}{2} b \cdot \frac{\sqrt{3}}{2} b\right) + 3(b \cdot \ell)$

$SA(b) = \frac{\sqrt{3}}{2} b^2 + \frac{81}{\sqrt{3}\, b}$

$SA'(b) = \sqrt{3}\, b - \frac{81}{\sqrt{3}\, b^2}$

$SA'(b) = 0$

$\Rightarrow \frac{81}{\sqrt{3}\, b^2} = \sqrt{3}\, b$

$\Rightarrow 81 = 3b^3$

$\Rightarrow 27 = b^3$

$\Rightarrow 3 = b$

$SA''(b) = \sqrt{3} + \frac{81 \cdot 2}{\sqrt{3}\, b^3} > 0$

$\boxed{\text{So } b = 3 \text{ is a minimum.}}$

4. (10 points)

(a) Estimate the definite integral $\int_0^1 \sqrt{1-x^2}\,dx$ by a Riemann sum using $\underline{n=4}$ rectangles and right endpoints as sample points. You don't need to simplify your answer; you may leave your answer as a sum of four terms.

$\Delta x = \frac{b-a}{n} = \frac{1}{n}$

$x_i^* = a + i\Delta x = \frac{i}{n}$

$f(x_i^*) = \sqrt{1-\left(\frac{i}{n}\right)^2}$

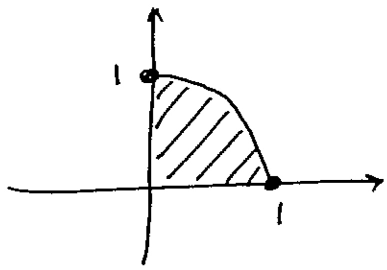
$\underline{n=4}$: $\Delta x = \frac{1}{4}$

$x_i^* = \frac{i}{4}$

$f(x_i^*) = \sqrt{1-\left(\frac{i}{4}\right)^2}$

$$\boxed{\sum_{i=1}^{4} \Delta x\, f(x_i^*) = \frac{1}{4}\left(\sqrt{1-\left(\frac{1}{4}\right)^2} + \sqrt{1-\left(\frac{2}{4}\right)^2} + \sqrt{1-\left(\frac{3}{4}\right)^2} + \sqrt{1-\left(\frac{4}{4}\right)^2}\right)}$$

(b) Evaluate $\int_0^1 \sqrt{1-x^2}\,dx$. Hint: Consider the shape of the region.



Region = $\frac{1}{4}$ of a circle

Area = $\frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi(1)$

$\boxed{= \frac{\pi}{4}}$

$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1 = r^2$

5. (10 points)

If $f(x) = \int_0^{\sin x} \sqrt{1+t^2}\, dt$ and $g(y) = \int_3^y f(x)\, dx$, find $g''(\pi)$.

By the Fundamental Theorem of Calculus,

$$g'(y) = f(y) = \int_0^{\sin(y)} \sqrt{1+t^2}\, dt .$$

Applying the Fundamental Theorem of Calculus again yields

$$g''(y) = \sqrt{1+(\sin(y))^2} \cdot \underbrace{\cos(y)}_{\text{Must use the chain rule.}}$$

Thus,

$$g''(\pi) = \sqrt{1+0} \cdot (-1)$$

$$\boxed{g''(\pi) = -1}$$

6. (15 points) Evaluate the following integrals. Express each answer as a single fraction.

(a) $\int_{-1}^{2}(3u-2)(u+1)\,du$

$$= \int_{-1}^{2} 3u^2 + u - 2\,du = u^3 + \frac{u^2}{2} - 2u \Big]_{u=-1}^{u=2}$$

$$= (8+2-4) - (-1+\tfrac{1}{2}+2)$$

$$= 6 - \frac{3}{2}$$

$$= \boxed{\frac{9}{2}}$$

(b) $\int_{1}^{4}\frac{2+x^2}{\sqrt{x}}\,dx = \int_1^4 (2+x^2)\,x^{-\frac{1}{2}}\,dx$

$$= \int_1^4 2x^{-\frac{1}{2}} + x^{3/2}\,dx$$

$$4^{5/2} = (4^{1/2})^5 = 2^5 = 32$$

$$= 4x^{1/2} + \frac{2}{5}x^{5/2}\Big]_{x=1}^{x=4}$$

$$= \left(8 + \frac{64}{5}\right) - \left(4 + \frac{2}{5}\right)$$

$$= \boxed{\frac{82}{5}}$$

(c) Remember that your answer should be a fraction (with no e or ln).

$$\int_0^{\sqrt{\ln 2}} xe^{x^2}\,dx$$

$u = x^2$ $\quad u(0) = 0$, $u(\sqrt{\ln 2}) = (\sqrt{\ln 2})^2 = \ln 2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int_0^{\ln 2} e^u \frac{du}{2} = \frac{1}{2}\int_0^{\ln 2} e^u\,du$$

$$= \frac{1}{2} e^u \Big]_{u=0}^{u=\ln 2}$$

$$= \frac{1}{2}[2-1]$$

$$= \boxed{\frac{1}{2}}$$

7. (15 points) Evaluate the following integrals.

(a) $\int \sec^2 x + 1\,dx$

$$\boxed{\tan(x) + x + C}$$

(b) $\int \cot(x)\,dx$

$$\int \frac{\cos(x)}{\sin(x)}\,dx \qquad u = \sin(x) \qquad du = \cos(x)\,dx \qquad \int \frac{1}{u}\,du = \ln|u| + c$$

$$\boxed{\int \cot(x)\,dx = \ln|\sin(x)| + c}$$

(c) $\int \frac{1}{(2x+5)^3}\,dx$ $= \frac{1}{2}\int \frac{1}{u^3}\,du = -\frac{1}{4}u^{-2} + c$

$u = 2x+5$

$du = 2dx$

$\frac{1}{2}du = dx$

$$\boxed{= -\frac{1}{4(2x+5)^2} + c}$$

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