MTH 142: Calculus II

Final Exam

December 17, 2017

NAME (please print legibly): ____

Your University ID Number: ____

Indicate your instructor with a check in the appropriate box:

Crossen	MW 9-10:15	
Zhong	MW 3:25-4:40	

- You have 3 hours to work on this exam.
- You are responsible for checking that this exam has all 18 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Show all work and justify all answers.
- Read the following Academic Honesty Statement and sign:

  I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

  Signature:____

Part A		
QUESTION	VALUE	SCORE
1	18	
2	10	
3	10	
4	16	
5	21	
TOTAL	75	

Part B		
QUESTION	VALUE	SCORE
1	20	
2	20	
3	10	
4	15	
5	10	
TOTAL	75	

Part A

1. (18 points) Consider the function (used throughout this problem)

$$g(x) = \frac{3x^2 - 3}{(x-3)^2}.$$

(a) Determine the following for $g(x)$.

Domain:

y**-intercept:**

x**-intercept(s):**

Vertical Asymptote(s):

Horizontal Asymptote(s):

(b) For the same $g(x)$, given that

$$g'(x) = \frac{6(1-3x)}{(x-3)^3},$$

determine the following for $g(x)$.

Open intervals on which $g(x)$ is increasing:

Open intervals on which $g(x)$ is decreasing:

local maximum value(s) of $g(x)$:

local minimum value(s) of $g(x)$:

(c) For the same $g(x)$, given that

$$g''(x) = \frac{36(x+1)}{(x-3)^4},$$

determine the following for $g(x)$.

Open intervals on which $g(x)$ is concave up:

Open intervals on which $g(x)$ is concave down:

Inflection point(s):

(d) Sketch the graph of $g(x)$.

2. (10 points) A box with a square base and an open top must have a volume of 1000 cm^3. The cost of the material for the base is $\$2/\text{cm}^2$ and the cost of the material for the sides is $\$1/\text{cm}^2$. Find the dimensions of the box that minimize the cost of materials.

3. (10 points) Calculate the volume generated by rotating the region bounded by the curves $y = \ln x$, $y = 0$, and $x = 2$, about the y-axis. You may use either method of washers/disks or cylindrical shells, but clearly indicate which method you use.

4. (16 points) Consider a water tank in the shape of an inverted cone (point down) with a radius of 2 meters, height of 7 meters, and a spout extending 2 meters above the top of the tank. (Recall the gravitational constant is $g = 9.8\ \mathrm{m/s^2}$ and the density of water is $1000\,\mathrm{kg/m^3}$)

(a) If the tank is only filled to a height of 5 meters with water, find an integral that represents the total work required to pump all of the water out of the spout. **(Do not evaluate the integral.)**

(b) If the tank is instead completely filled with water, find an integral that represents the total work required to pump water out of the spout until **ONLY 2 METERS OF WATER REMAIN IN THE TANK**. **(Do not evaluate the integral.)**

5. (21 points) Evaluate the following integrals.

(a) $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

(b) $\displaystyle\int \tan^3 x \sec^4 x \, dx$

(c) $\int e^{2y} \cos y \, dy$

Part B

1. (20 points) Evaluate the following integrals:

(a) $\displaystyle\int \frac{\sqrt{x^2-9}}{2x}\,dx$

(b) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

2. (20 points) Evaluate the following integrals:

(a) $\displaystyle\int \frac{x^2-1}{x^3+x}\,dx.$

(b) $\displaystyle\int \frac{x^3-3x^2-x-1}{x^2-2x-3}\,dx$

3. (10 points) Set up the partial fraction decomposition for the following integral in terms of variables but do not solve for those variables.

$$\int \frac{1}{(x^3 + x^2 + x)(x^3 - x)(x^2 + x)(x^2 + x + 1)}\, dx$$

4. (15 points) Determine if each integral is convergent or divergent. Evaluate those that are convergent. Show all your work.

(a) $\displaystyle\int_1^\infty \frac{\ln x}{x}\, dx$

(b) $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\, dx$

5. (10 points) Find the arc length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \leq x \leq 2.$$

Blank page for scratch work

Formula Sheet

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$
- $\sin(2x) = 2 \sin x \cos x$
- $\sin^2 x = \dfrac{1 - \cos(2x)}{2}$
- $\cos^2 x = \dfrac{1 + \cos(2x)}{2}$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\sin x \cos y = \dfrac{\sin(x - y) + \sin(x + y)}{2}$
- $\sin x \sin y = \dfrac{\cos(x - y) - \cos(x + y)}{2}$
- $\cos x \cos y = \dfrac{\cos(x - y) + \cos(x + y)}{2}$