Math 142: Midterm 2

University of Rochester

November 8, 2022

Name: Solutions

UR ID: ______

UR E-mail: ______

Section	"X" your class time
MW 9 AM	
MW 3:25 PM	

- You are allowed one page, single-sided of notes. No other resources are permitted.
- The exam questions are on pages 2-11 of this packet.
- Each part of each question is on its own page. All work you want graded for that problem should be contained entirely on that page, unless:
- If you need more space on a problem, use the **Scratch work** pages at the end of the exam, and make sure to make a note on the problem page that you are doing so.
- **Do not tear off the scratch work pages.**
- Copy and sign the Honor Pledge: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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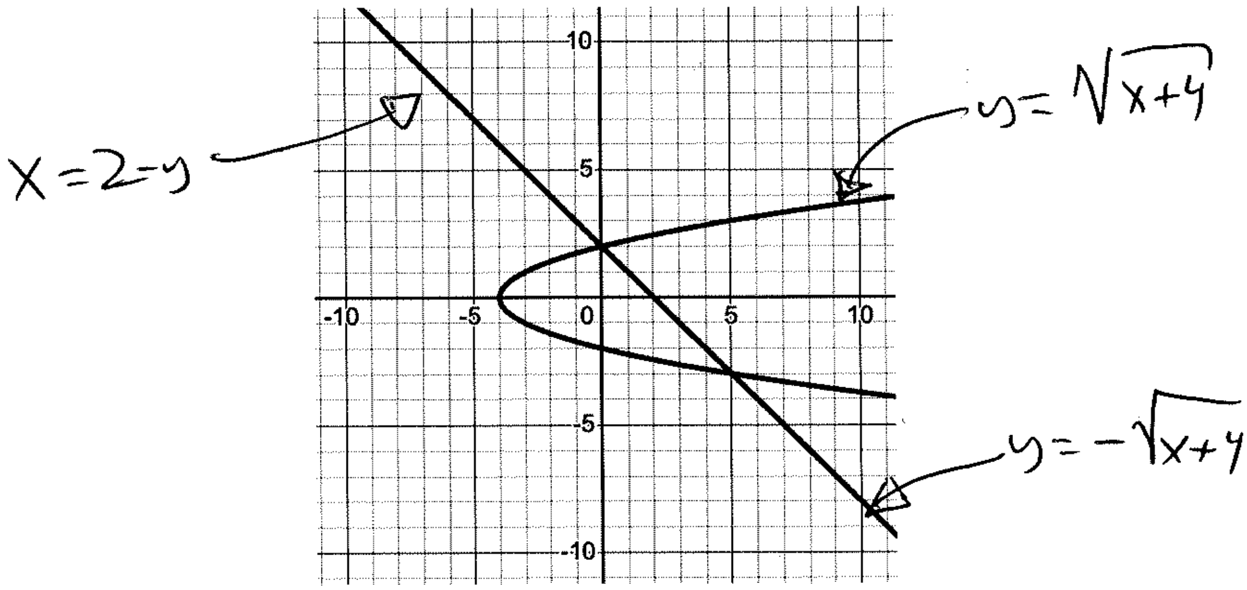
Signature: ______

Question:	1	2	3	4	5	6	Total
Points:	20	20	10	30	10	10	100

1. Consider the region R in the xy-plane bounded by the curves $x = y^2 - 4$ and $y = 2 - x$, shown below.



(a) (10 points) Write down (but do not evaluate) an integral formula for the area of R with respect to x.

$$x = y^2 - 4 \Rightarrow y = \pm\sqrt{x+4}$$

$$A = \int_{-4}^{0} \sqrt{x+4} - (-\sqrt{x+4})\, dx + \int_0^5 (2-x) - (-\sqrt{x+4})\, dx$$

ANSWER:

$$\boxed{\int_{-4}^{0} 2\sqrt{x+4}\, dx + \int_0^5 (2-x) + \sqrt{x+4}\, dx}$$

(b) (10 points) Write down (but do not evaluate) an integral formula for the area of R with respect to y.

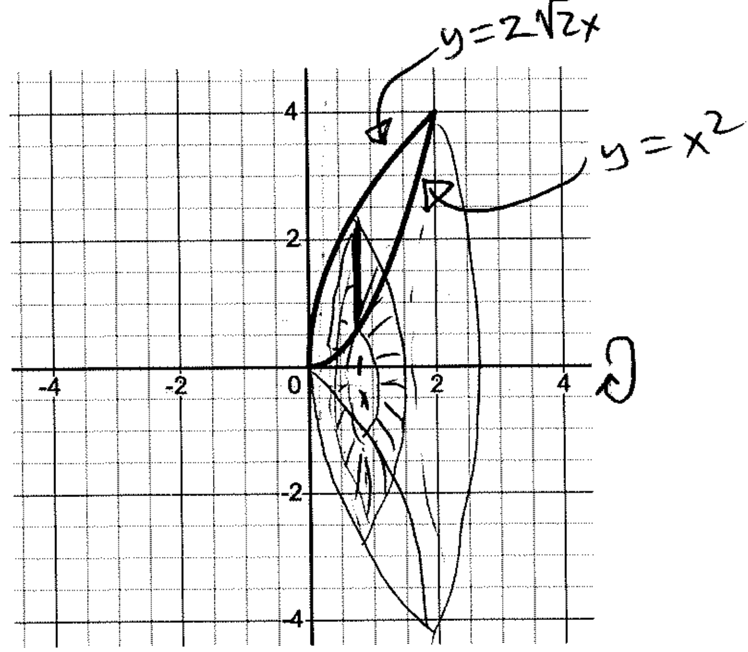
$$y = 2 - x$$

$$\Longleftrightarrow x = 2 - y$$

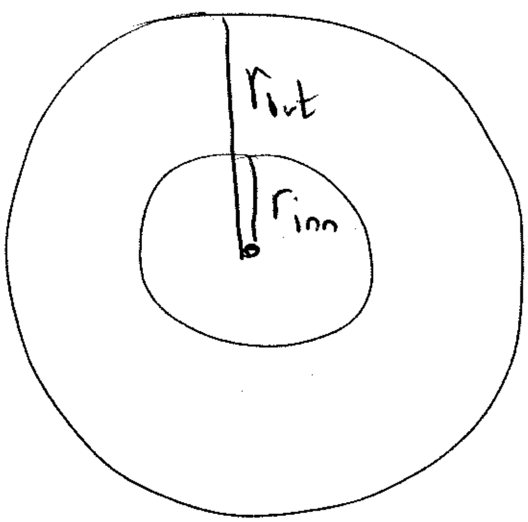
ANSWER:

$$\boxed{\int_{-3}^{2} (2-y) - (y^2 - 4)\, dy}$$

2. Let R be the first quadrant region bounded by the curves $y = x^2$ and $y = 2\sqrt{2x}$, shown below:



(a) (10 points) Using the **washer method**, write down (but do not evaluate) an integral for the volume of the solid obtained by revolving R about the x-axis.



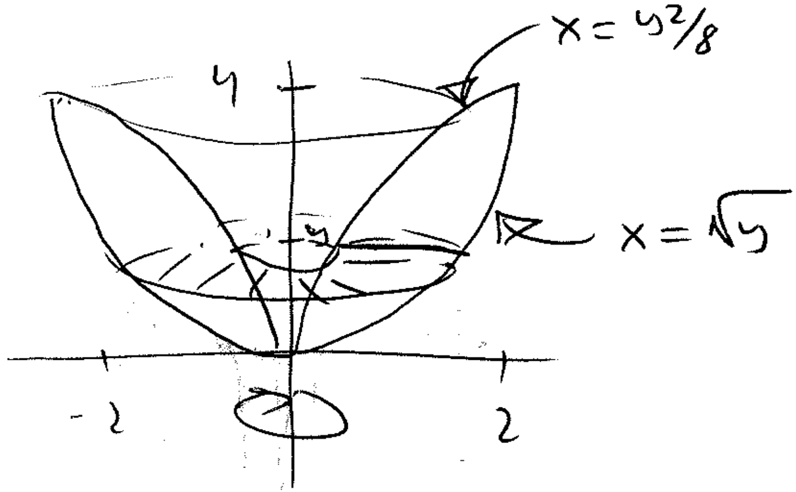
$r_{inn} = x^2$

$r_{out} = 2\sqrt{2x}$

ANSWER:

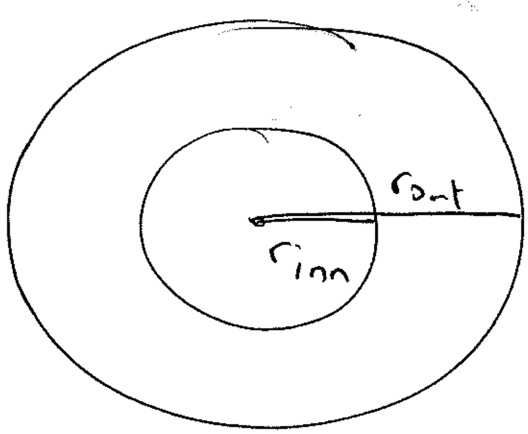
$$\int_0^2 \pi\left(\left(2\sqrt{2x}\right)^2 - \left(x^2\right)^2\right) dx$$

(b) (10 points) Using the **washer method**, write down (but do not evaluate) an integral for the volume of the solid obtained by revolving R about the y-axis.



$$y = 2\sqrt{2x} \iff y^2 = 2^2 \cdot 2x = 8x$$

$$\Rightarrow x = \frac{y^2}{8}$$



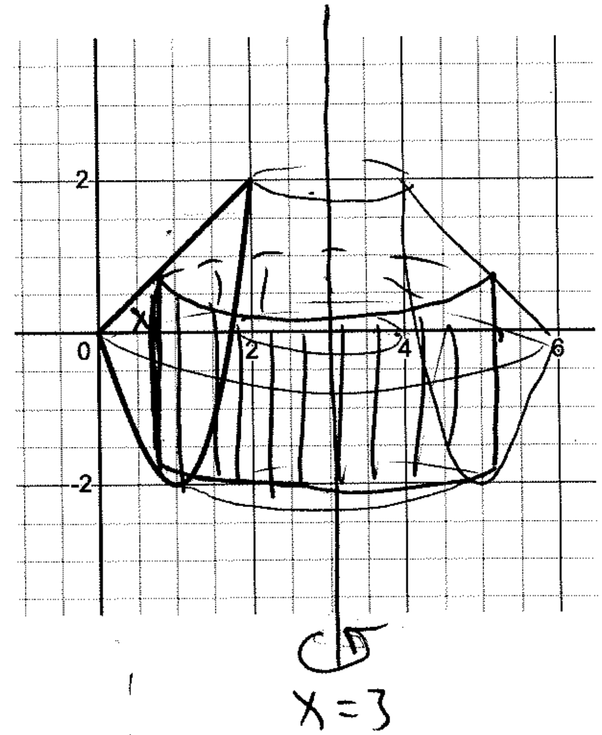
$$r_{inn} = \frac{y^2}{8}$$

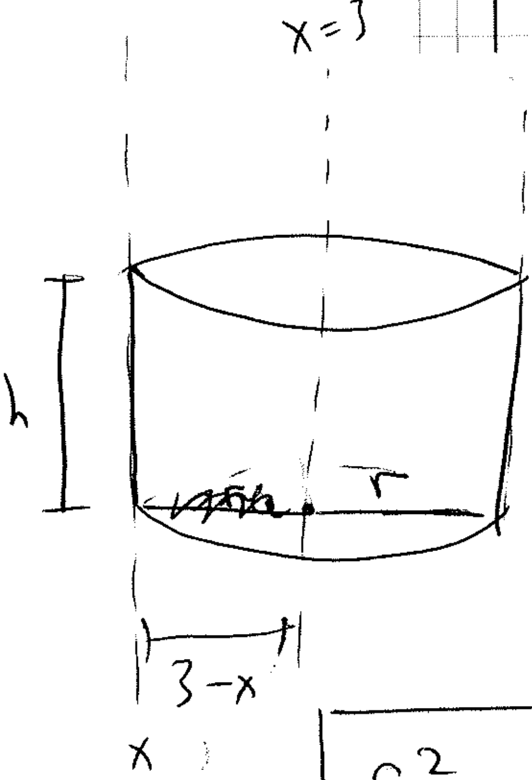
$$r_{out} = \sqrt{y}$$

ANSWER:

$$\boxed{\int_0^4 \pi\left((\sqrt{y})^2 - \left(\frac{y^2}{8}\right)^2 \right) dy}$$

3. (10 points) A region R bounded by the curves $y = x$ and $y = x^3 - 3x$ is shown below. Using the **shell method**, write down (but do not evaluate) an integral for the volume V of the solid S obtained by revolving R about the vertical line $x = 3$.





$r = 3 - x$

$h = x - (x^3 - 3x)$

ANSWER: $\boxed{\int_0^2 2\pi(3-x)\left(x-(x^3-3x)\right)}$

4. Calculate the following integrals.

(a) (10 points) $\int (1+x^2)^{-1} e^{\arctan x}\, dx$

$u = \arctan(x)$

$du = \frac{1}{1+x^2}\, dx = (1+x^2)^{-1}\, dx$

$= \int e^u\, du = e^u + C$

$= e^{\arctan(x)} + C$

ANSWER:

$\boxed{e^{\arctan(x)} + C}$

(b) (10 points) $\int_1^6 2x\sqrt{x+3}\,dx$

$u = x+3$

$du = dx, \quad x = u-3$

$$= \int_4^9 2(u-3)\sqrt{u}\,du$$

$$= 2\int_4^9 u^{3/2}\,du - 6\int_4^9 u^{1/2}\,du$$

$$= 2\cdot\frac{2}{5}u^{5/2}\Big|_4^9 - 6\cdot\frac{2}{3}u^{3/2}\Big|_4^9$$

$$= \frac{4}{5}\left(9^{5/2} - 4^{5/2}\right) - 4\left(9^{3/2} - 4^{3/2}\right)$$

$$= \frac{4}{5}(243 - 32) - 4(27 - 8)$$

$$= \frac{4}{5}(211) - 4\cdot 19 \ldots$$

ACCEPTED

ANSWER:

$$\boxed{\frac{4}{5}\left(9^{5/2} - 4^{5/2}\right) - 4\left(9^{3/2} - 4^{3/2}\right)}$$

(c) (10 points) $\int \frac{(\ln(x))^2+1}{x}\,dx$

$$= \int \frac{(\ln(x))^2}{x}\,dx + \int \frac{1}{x}\,dx$$

$$= \int \frac{(\ln(x))^2}{x}\,dx + \ln|x| + C$$

$u = \ln(x)$, $du = \frac{1}{x}\,dx$

$$= \int u^2\,du + \ln|x| + C$$

$$= \frac{u^3}{3} + \ln|x| + C$$

$$= \frac{(\ln(x))^3}{3} + \ln|x| + C$$

ANSWER:

$$\boxed{\frac{(\ln(x))^3}{3} + \ln|x| + C}$$

5. (10 points) Let $g(x) = \int_1^{x^2+1} \cos\left(t - \frac{1}{t}\right)\, dt$. Find $g'(x)$.

$$f(x) = \cancel{\int_1^{x^2+1}} \int_1^{x} \cos\left(t - \frac{1}{t}\right) dt$$

$$h(x) = x^2+1$$

$$\Rightarrow g(x) = f(h(x))$$

$$\Rightarrow g'(x) = f'(h(x)) \cdot h'(x)$$

$$= \left(\cos\left(h(x) - \frac{1}{h(x)}\right)\right) \cdot h'(x)$$

ANSWER:

$$\boxed{\left(\cos\left(x^2+1 + \frac{1}{x^2+1}\right)\right) \cdot 2x}$$

6. (10 points) Suppose that $\int_1^2 f(x)\,dx = 5$. Find $\int_0^{\pi/2} \frac{f(\sin(x)+1)\cos(x)}{2}\,dx$.

$$\int_0^{\frac{\pi}{2}} \frac{f(\sin(x)+1)\cos(x)}{2}\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} f(\sin(x)+1)\cos(x)\,dx$$

$$u = \sin(x)+1$$
$$du = \cos(x)$$
$$x = 0 \Rightarrow u = 1$$
$$x = \frac{\pi}{2} \Rightarrow u = 2$$

$$= \frac{1}{2}\int_1^2 f(u)\,du$$

$$= \frac{1}{2}\cdot 5$$

ANSWER:

$5/2$

Scratch work (first page) — DO NOT REMOVE