Math 142: Midterm 1

University of Rochester

October 4, 2022

Name: _____

UR ID: _____

UR E-mail: _____

Section	"X" your class time
MW 9 AM	
MW 3:25 PM	

- You are allowed one page, single-sided of notes. No other resources are permitted.

- The exam questions are on pages 2-11 of this packet.

- Each part of each question is on its own page. All work you want graded for that problem should be contained entirely on that page, unless:

- If you need more space on a problem, use the **Scratch work** pages at the end of the exam, and make sure to make a note on the problem page that you are doing so.

- **Do not tear off the scratch work pages.**

- Copy and sign the Honor Pledge: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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_____

Signature: _____

Question:	1	2	3	4	5	6	Total
Points:	30	10	15	15	15	15	100

1. Consider the function $f(x)$ defined by

$$f(x) = \frac{x}{x^2 - 1}.$$

The first and second derivatives of $f(x)$ are

$$f'(x) = -\frac{1 + x}{(x^2 - 1)^2} \quad \text{and} \quad f''(x) = \frac{2x(3 + x)^2}{(x^2 - 1)^3}.$$

(a) (2 points) What is the domain of $f(x)$?

ANSWER:

(b) (2 points) List the x-intercepts of $f(x)$.

ANSWER:

(c) (2 points) List the y-intercepts of $f(x)$.

ANSWER:

(d) (2 points) Find all the vertical asymptotes of $f(x)$, or explain why none exist.

ANSWER:

(e) (2 points) Find all the horizontal asymptotes of $f(x)$, or explain why they do not exist.

ANSWER:

(f) (2 points) Find all the intervals where $f(x)$ is **increasing**.

ANSWER:

(g) (2 points) Find all the intervals where $f(x)$ is **decreasing**.

ANSWER:

(h) (2 points) Find all the critical numbers of $f(x)$, or explain why none exist.
ANSWER:

(i) (2 points) Find all the intervals where $f(x)$ is **concave up**.
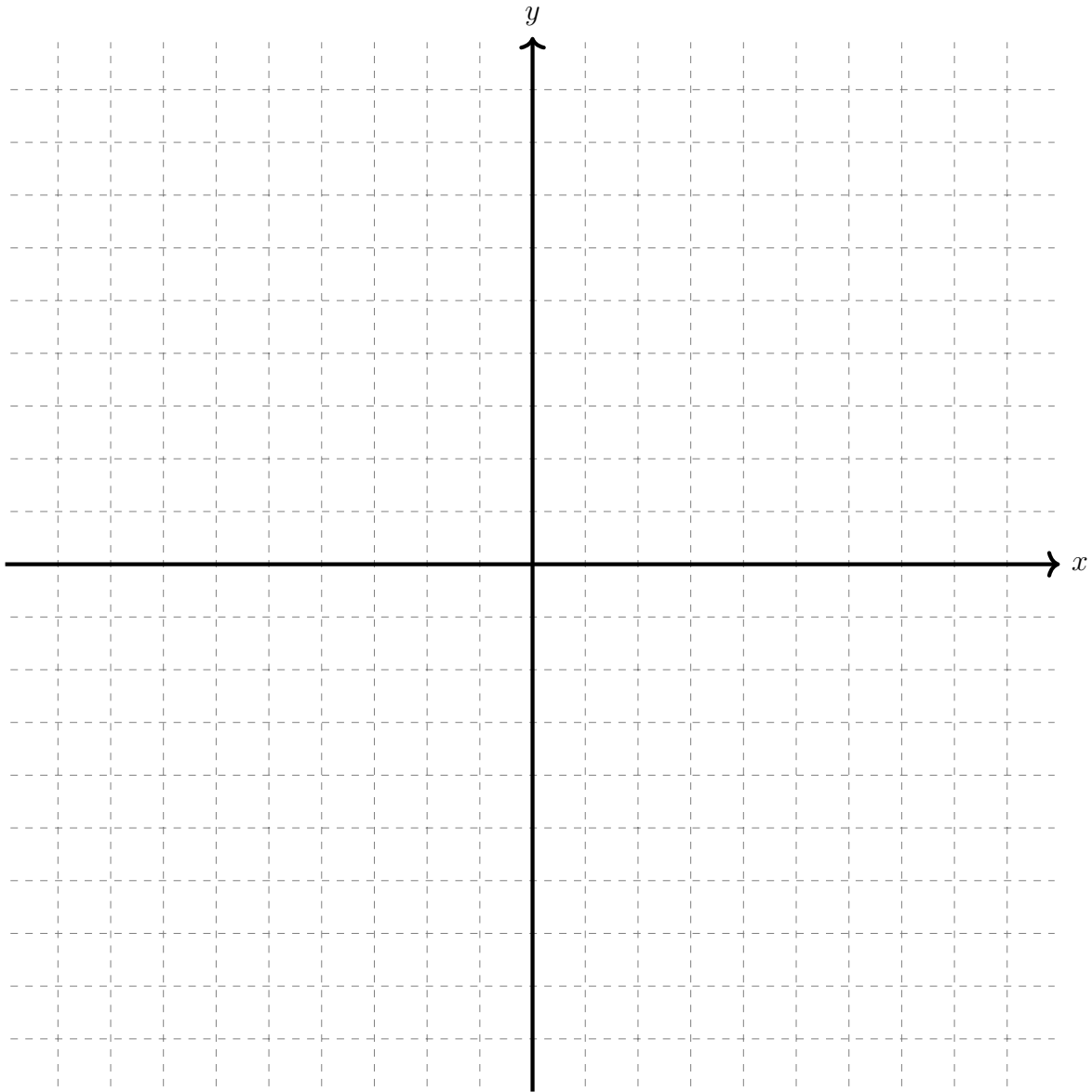

ANSWER:

(j) (2 points) Find all the intervals where $f(x)$ is **concave down**.

ANSWER:

(k) (2 points) Find all the inflection points of $f(x)$, or explain why none exist.
ANSWER:

(l) (8 points) Use your work from parts (a)-(k) to graph $f(x)$ below. Note that you may scale the axes how you like (i.e. prioritize a good sketch over using 1 tick mark to represent 1 unit along an axis).

2. (10 points) Find the absolute minimum and maximum values of $f(x) = x^4 - 2x^2$ on the interval $[-1, 2]$.

ANSWER:

3. (15 points) If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest volume of the box. Make sure to completely justify your answer.

ANSWER:

4. Compute the following indefinite integrals:

(a) (5 points) $\int \dfrac{3}{x^2} + e^x + \sec^2 x \, dx$

ANSWER:

(b) (5 points) $\int \left(x + \dfrac{1}{x} \right) (2x + 1) \, dx$

ANSWER:

(c) (5 points) $\displaystyle\int \frac{2}{1+x^2} + \frac{1+x^2}{x^2}\, dx$

ANSWER:

5. (15 points) Consider a particle on the x-axis which starts to move from the origin at $t = 0$, i.e. if $x(t)$ is the function indicating the location of the particle at time t, then $x(0) = 0$. If the velocity of this particle at time t is given by the function $v(t) = t^2 + t + e^t$, find the location of the particle at $t = 2$.

ANSWER:

6. (15 points) Find the left endpoint Riemann sum L_4 (i.e. using 4 subintervals) for the function $f(x) = \cos(x) + 1$ above the interval $[0, 2\pi]$. **Simplify your answer as much as possible.** A graph of $f(x)$ is given below.
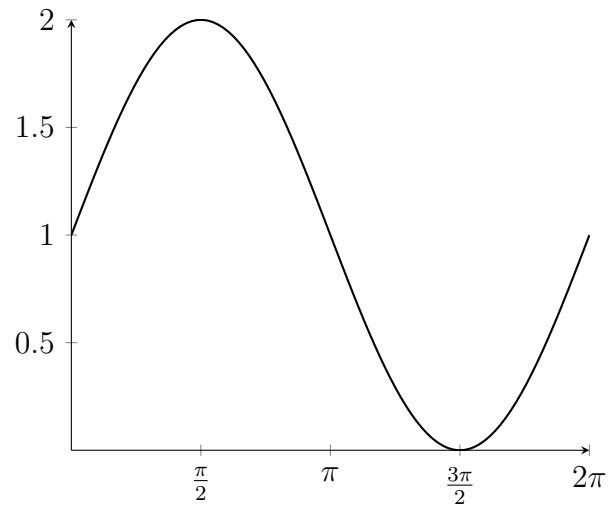
ANSWER:

Scratch work (first page) — DO NOT REMOVE

Scratch work (second page) — DO NOT REMOVE

Scratch work (third page) — DO NOT REMOVE