Math 142: Final Exam

University of Rochester

December 18, 2022

Name: _____

UR ID: _____

UR E-mail: _____

Section	"X" your class time
MW 9 AM	
MW 3:25 PM	

- You are allowed one page, single-sided of notes. No other resources are permitted.
- The exam questions are on pages 2-21 of this packet. **Part A consists of problems 1-6. Part B consists of problems 7-11.**
- Each part of each question is on its own page. All work you want graded for that problem should be contained entirely on that page, unless:
- If you need more space on a problem, use the **Scratch work** pages at the end of the exam, and make sure to make a note on the problem page that you are doing so.
- **Do not tear off the scratch work pages.**
- Copy and sign the Honor Pledge: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

_____

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Signature: _____

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	25	10	20	15	10	20	30	20	10	30	10	200

Part A

1. Consider the following function and its derivatives:

$$f(x) = \frac{1}{1-x^2} \qquad f'(x) = \frac{2x}{(1-x^2)^2} \qquad f''(x) = -\frac{2(3x^2+1)}{(x^2-1)^3}.$$

(a) (2 points) What is the domain of $f(x)$? Write it in interval notation.
ANSWER:

(b) (2 points) What are the x and y intercepts of $f(x)$ (if any)?
ANSWER:

(c) (2 points) What are the horizontal and vertical asymptotes of $f(x)$ (if any)?
ANSWER:

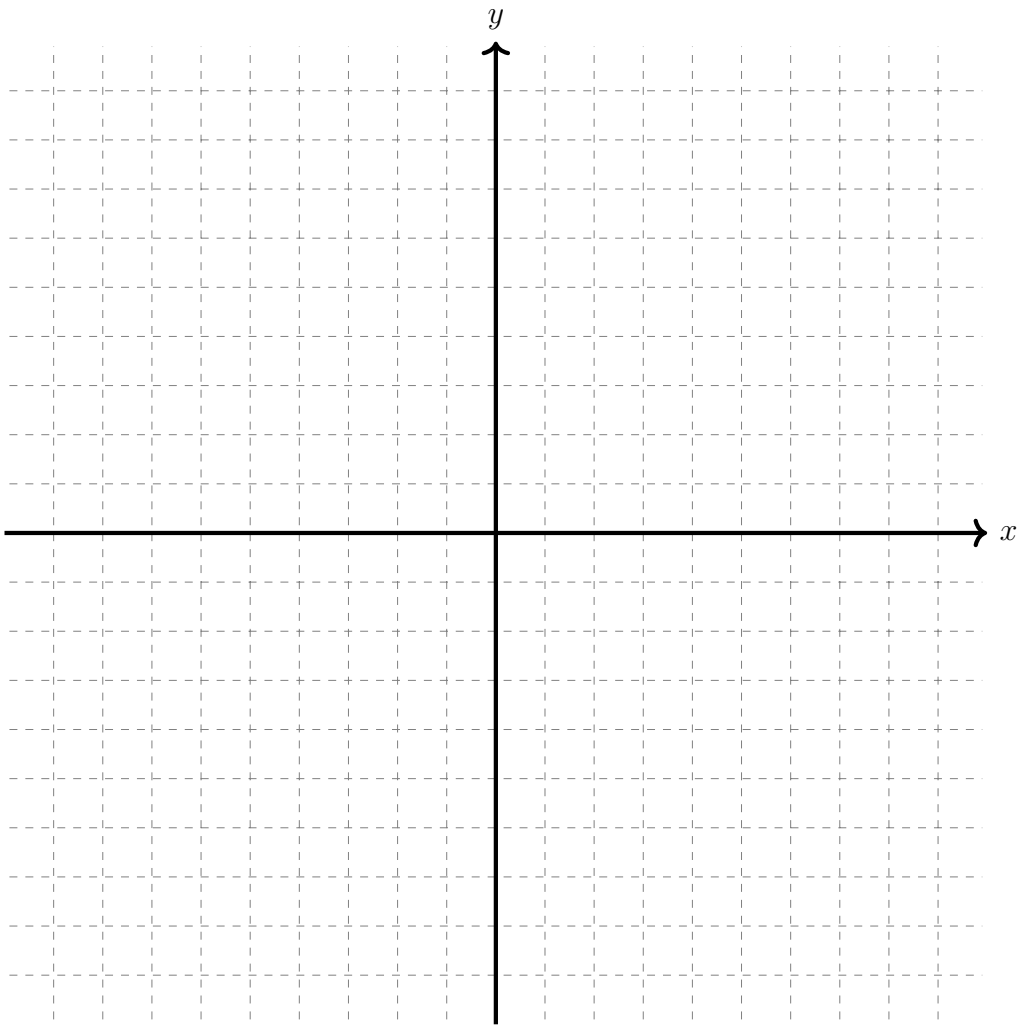
(d) (3 points) On which intervals is $f(x)$ increasing, and on which intervals is $f(x)$ decreasing?
ANSWER:

(e) (3 points) What are the local extrema of $f(x)$, if any?
ANSWER:

(f) (3 points) What are the intervals of concavity of $f(x)$?
ANSWER:

(g) (3 points) What are the inflection points of $f(x)$, if any?
ANSWER:

(h) (7 points) Sketch the graph of $f(x)$. You may scale the axes how you like (i.e. prioritize a good sketch over using 1 tick mark to represent 1 unit along an axis).



2. (10 points) What is the area of the largest rectangle in the first quadrant of the xy-plane with one corner at the origin and opposite corner on the graph of $f(x) = 1 - x^2$?

ANSWER:

3. Consider a particle moving on the real line, whose acceleration as a function of time is given by $a(t) = \dfrac{2}{(t+1)^3}$ for $t \geq 0$. Moreover, assume that the initial position and velocity of the particle are given by $x(0) = 0$ and $v(0) = -1$, respectively.

   (a) (10 points) What is the location of the particle at time $t = 1$?

ANSWER:

(b) (10 points) What is the velocity of the particle at time $t = 1$?

ANSWER:

4. Compute the following integrals.

   (a) (5 points) $\int 2xe^{x^2+1}\,dx$

ANSWER:

(b) (5 points) $\int t^2\sqrt{t-1}\,dt$

ANSWER:

(c) (5 points) $\displaystyle\int \frac{2\cos(x)}{1+\sin^2(x)}\,dx$

ANSWER:

5. (10 points) Find the area enclosed by the graphs of $y = |x|$ and $y = 2 - x^2$.

ANSWER:

6. Let R be the region in the plane above the x-axis and below the graph of $y = \sin(x)$ from $x = 0$ to $x = 2\pi$.

   (a) (10 points) Write down (**but do not calculate**) an integral expressing the volume of the solid region obtained by rotating R around the horizontal line $y = 2$.

ANSWER:

   (b) (10 points) Write down (**but do not calculate**) an integral expressing the volume of the solid region obtained by rotating R around the y-axis.

ANSWER:

Part B

7. Compute the following integrals.

   (a) (10 points) $\int t^2 \ln(2t)\, dt$

ANSWER:

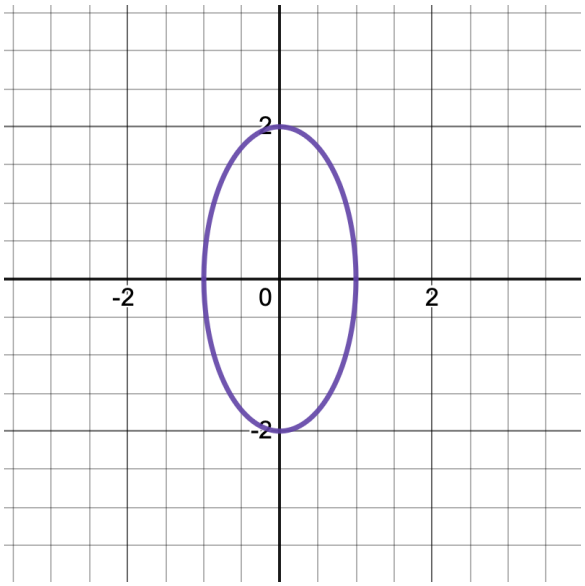
(b) (10 points) $\int \sin^3(x)\cos^2(x)\,dx$

ANSWER:

(c) (10 points) $\int_0^1 x^2 e^{-x}\,dx$

ANSWER:

8. Consider the ellipse $x^2 + \dfrac{y^2}{4} = 1$, pictured below. Let R be the first quadrant region enclosed by the ellipse and the coordinate axes.



(a) (10 points) Write the area A of R as an integral.

ANSWER:

(b) (10 points) Use trigonometric substitution to compute A.

ANSWER:

9. (10 points) Find $\displaystyle\int \frac{x^4}{x^3 - 2x^2 + x}\, dx$.

ANSWER:

10. Determine if the following improper integrals converge or diverge. Justify your work completely, with correct limit work (no plugging in ∞ as an argument into functions!) when needed.

(a) (10 points) $\int_1^\infty e^{-x}\,dx$

ANSWER:

(b) (10 points) $\displaystyle\int_0^{100} \frac{1}{x^{1/5}}\,dx$

ANSWER:

(c) (10 points) $\displaystyle\int_1^\infty \frac{x^2}{x^6 + \ln(x)}\, dx$

ANSWER:

11. (10 points) Find the length of the curve given by $y = 2x^{3/2}$ for $0 \leq x \leq 1$.

ANSWER:

Scratch work (first page) — DO NOT REMOVE

Scratch work (second page) — DO NOT REMOVE

Scratch work (third page) — DO NOT REMOVE