Class 4

$\sin(-x) = -\sin(x)$

$\cos x = \cos(-x)$

graph of cos

e^x domain $(-\infty, \infty)$

Range $(0, +\infty)$

$f(-x) = f(x)$

$f(-x) = -f(x)$

$\ln x$ domain $(0, +\infty)$ Range $(-\infty, \infty)$

change base of log $\boxed{\log_a b} = \frac{\ln b}{\ln a}$

$\boxed{\log_3 10} = \frac{\ln 10}{\ln 3}$

Not only can be changed to e

$\frac{3}{2} = \boxed{\log_9 27} = \frac{\log_3 27}{\log_3 9} = \frac{3}{2}$

$27 = 3^3 = (3^2)^{\frac{3}{2}} = 9^{\frac{3}{2}}$

$3 = 2 \cdot \frac{3}{2}$

① even function

odd function

a) $f(x) = 2x^5 - 3x^2$ x odd x even

$f(-x) = 2(-x)^5 - 3(-x)^2$

$= -2x^5 - 3x^2$

b) $f(x) = x^3 - x^7$ odd

$f(-x) = (-x)^3 - (-x)^7$

$= -x^3 + x^7$

c) $f(x) = e^{-x^2}$ even

$f(-x) = e^{-(-x)^2} = e^{-x^2} \quad f(x) = f(-x)$

④ $e^{2x} = 3$

$\ln x^2 = 5$

$\tan^{-1}(3x^2) = \frac{\pi}{4}$

$\ln x + 1 = \ln(5+x) - 4$

② $f(x) = \ln x \quad g(x) = x^2 - 9$

$f \circ g = \ln(x^2 - 9) \quad (-\infty, -3) \cup (3, +\infty)$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$(x-3) < (x+3)$

$\boxed{x > 3}$

$\boxed{x^2 - 3}$

$g \circ f = (\ln x)^2 - 9 \quad (0, +\infty)$

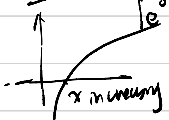
$f \circ f = \ln(\ln x)$

$\boxed{x > 1} \Leftarrow x > 0$

$\boxed{\ln x} > 0$

$\ln 1 = 0$

$\boxed{e^0 = 1}$



x increasing y increasing

$\ln x$

$(x-a)(x-b) > 0$

e^x Domain: $\mathbb{R}$

Range: $(0, +\infty)$

0 not included

Domain: $(0, +\infty)$

0 not included

Range: $\mathbb{R}$

⑤ find inverse function of $f(x) = \frac{2x+3}{1-5x}$

$y = \frac{2x+3}{1-5x}$

$y(1-5x) = 2x+3$

$y - 5xy = 2x + 3$

$y - 3 = 2x + 5xy$

$y - 3 = x(2 + 5y)$

$x = \frac{y-3}{2+5y}$

$f^{-1}(x) = \frac{x-3}{2+5x}$

③ a) $e^{2\ln 5} = (e^{\ln 5})^2 = 5^2 = 25$

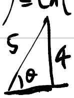
b) $\log_6 4 + \log_6 54 = \log_6(4 \times 54) = \log_6(216) = \log_6(6^3) = 3$

$216 = 6 \times 36 = 6 \times 6 \times 6 = 6^3$

③ $\tan(\arcsin(\frac{4}{5})) = \tan(\theta) = \frac{4}{3}$

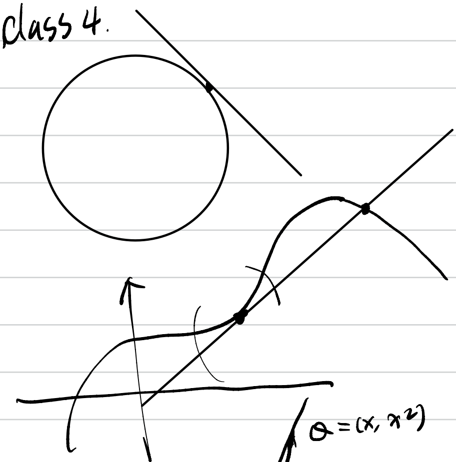
④ $\ln \frac{1}{e^3} = \ln((e^3)^{-1}) = \ln(e^{-3}) = -3$

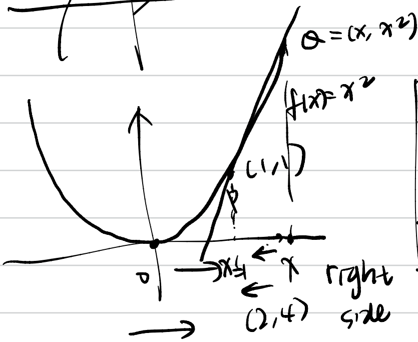
$\arcsin(\frac{4}{5}) = \theta$



$\sqrt{25 - 16} = \sqrt{9} = 3$

class 4.





x	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
	⋮
	2 ?

$$\frac{1.1^2-1}{1.1-1} = \frac{1.21-1}{0.1} = \frac{0.21}{0.1} = 2.1$$

$$m_2 = \frac{4-1}{2-1} = 3$$

$$m_{1.5} = \frac{1.5^2-1}{1.5-1} = \frac{2.25-1}{1.5-1} = \frac{1.25}{0.5} = 2.5$$

x	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.98	1.98
0.999	1.999
⋮	⋮

2

$$\frac{0^2-1}{0-1} = \frac{-1}{-1} = 1$$

$$\frac{0.5^2-1}{0.5-1} = \frac{0.75}{0.5} = 1.5$$

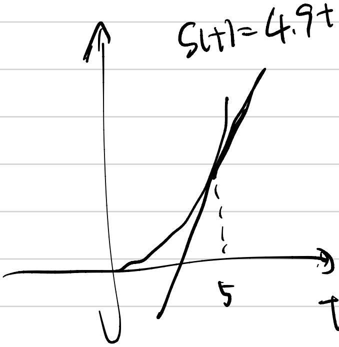
g=9.8

$S(t) = \frac{gt^2}{2}$

$= 4.9t^2$ try to the velocity when t=5

average velocity = $\frac{\text{distance it travel}}{\text{time it used}}$

$S(t)=4.9t^2$



t=5 ~ t=5.1

$\frac{S(5.1)-S(5)}{0.1}$ ← average

$\frac{S(5.01)-S(5)}{0.01}$

x	f(x)
0.5	0.6667
0.9	0.5263
0.99	0.502
0.999	0.560025

x	f(x)
1.5	0.4
1.1	0.476...
1.01	0.49...
1.001	0.499875

0.5

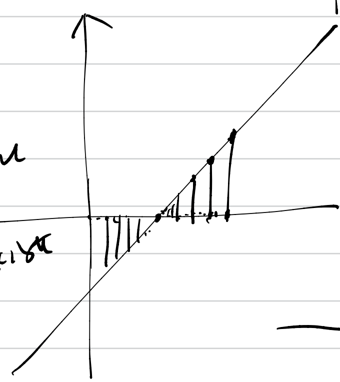
limit of function

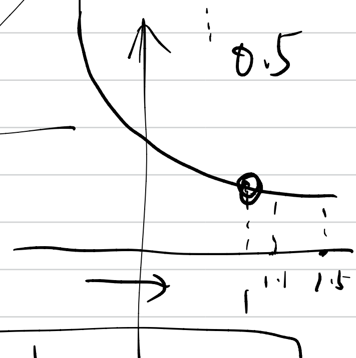
$f(x)=x-1$

let $x \to 1$

$f(x) \to 0$

f(1) not define
limit exist





$f(x) = \frac{x-1}{x^2-1}$ $x=1.1$
1.01
1.001

$f(1) = \frac{0}{0}$ not defined ?

$$\boxed{\lim_{x\to 1}\frac{x-1}{x^2-1} = 0.5}$$

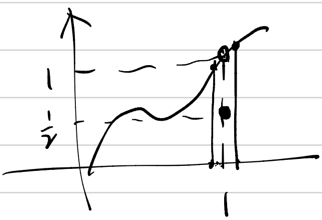
f(x) is defined when x is near number a

$\lim_{x\to a} f(x) = L$ if we can make the value of f(x) arbitrarily close to L by restricting x to be sufficiently closed to a [but not equal to a]

$\boxed{\frac{x-1}{x^2-1}}$

f(x) f(a) exist

$\lim_{x\to a} f(x) \neq f(a)$



$\lim_{x\to 1} f(x) = 1$

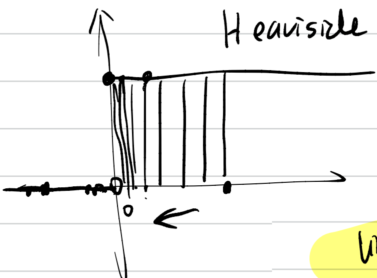
$f(1) = \frac{1}{2}$

$f(1) \neq \lim_{x\to 1} f(x)$

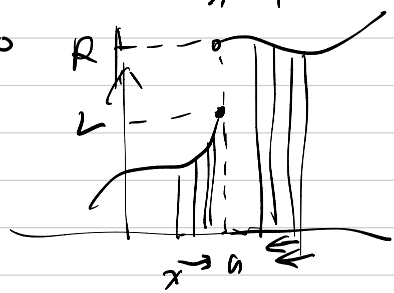
$\boxed{\lim_{x\to a^-} f(x) = L}$

$x < a$

Heaviside function



$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$



$\lim_{x\to 0^+} H(x) = 1$

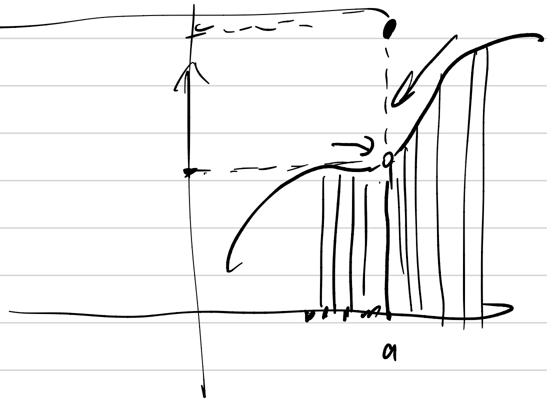
$\lim_{x\to 0^-} H(x) = 0$

$\lim_{x\to a^+} f(x) = R$

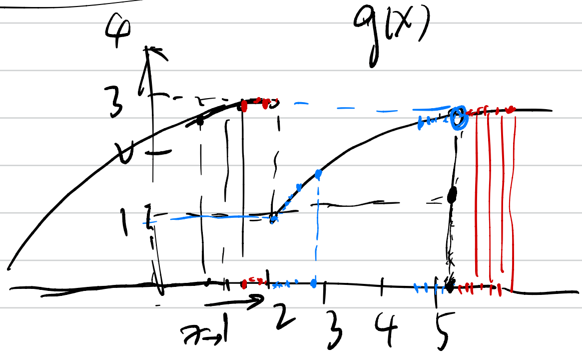
here x is $> a$

right hand limit of f(x)

$$\boxed{\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L} \iff \boxed{\lim_{x\to a} f(x) = L}$$



$f(a) = L$



$\lim_{x\to 2^-} g(x) = 3$

$\lim_{x\to 2^+} g(x) = 1$

$\lim_{x\to 2} g(x)$ doesn't exist

since $3 \neq 1$

$\lim_{x\to 5^-} g(x) = 3$

$\lim_{x\to 5^+} g(x) = 3$ $\quad g(5) = 1$

example that limit doesn't exist

$\sin\left(\frac{\pi}{x}\right)$ when $x \to 0$

$x=1 \quad \sin(\pi)=0$

$x=0.1 \quad \sin\left(\frac{\pi}{0.1}\right)=\sin(10\pi)=0$

$x=0.01 \quad \sin\left(\frac{\pi}{0.01}\right)=\sin(100\pi)=0$

$\vdots$

$x=0.00001 \quad \sin\left(\frac{\pi}{0.00001}\right)=0$

$x=\left(\frac{2}{1}\right) \quad \sin\left(\frac{\pi}{2}\right)=1$

$x=\left(\frac{2}{5}\right) \quad \sin\left(\frac{\pi}{\frac{2}{5}}\right)=\sin\left(\frac{5\pi}{2}\right)$

$=\sin\left(2\pi+\frac{\pi}{2}\right)$

$=\sin\left(\frac{\pi}{2}\right)=1$

$x=\left(\frac{2}{9}\right) \quad \sin\left(\frac{\pi}{x}\right)=\sin\left(\frac{9\pi}{2}\right)$

$=\sin\left(4\pi+\frac{\pi}{2}\right)$

$=1$

$x=\left(\frac{2}{13}\right)$

$\sin\left(\frac{\pi}{x}\right)=\sin\left(\frac{13\pi}{2}\right)$

$=\sin\left(6\pi+\frac{1}{2}\pi\right)$

$=1$

$\vdots$

$x=\frac{2}{4k+1} \qquad f(x)=\sin\left(\frac{\pi}{x}\right)$

$x=\frac{2}{4k-1} \qquad x=\frac{2}{3} \qquad x=\frac{2}{7} \qquad x=\frac{2}{11}$

$\sin\left(\frac{\pi}{x}\right)=\sin\left(\frac{3\pi}{2}\right)=-1 \qquad \sin\left(\frac{\pi}{x}\right)=\sin\left(\frac{7\pi}{2}\right)=-1$

$\sin\left(\frac{\pi}{x}\right)=\sin\left(\frac{11\pi}{2}\right)=-1$

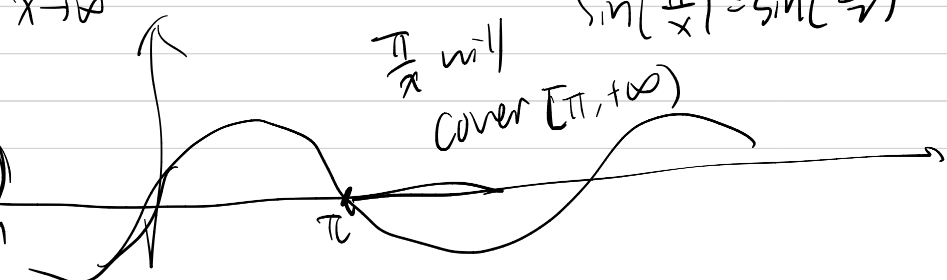
$x \to \infty$

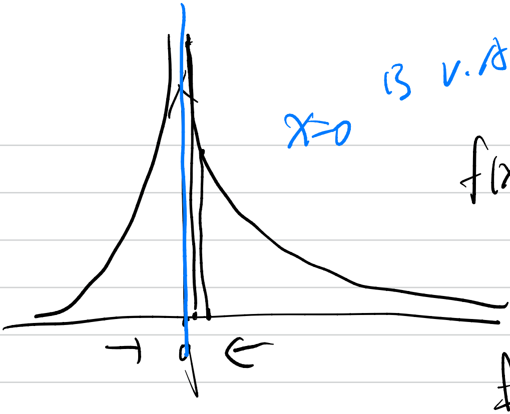
$\frac{\pi}{x}$ will cover $[\pi,+\infty)$

$f(x)=\sin x$

$g(x)=\sin\left(\frac{\pi}{x}\right)$

$0<x\le 1$





$x=0$ is v.a

Vertical asymptote

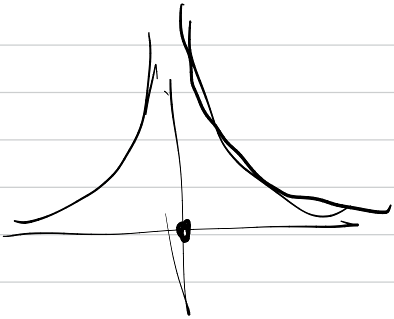
$f(x)=\frac{1}{x^2}$ ① $\lim_{x\to 0} f(x)=+\infty$ is not a number

$x\to 0$

$g(x)=\begin{cases}\frac{1}{x^2} & x\neq 0\\ 0 & x=0\end{cases}$

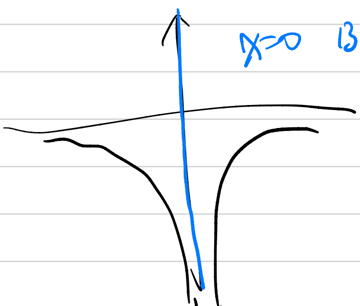
$\cancel{f(0)=\frac{1}{0}}$

$\lim_{x\to a} f(x)=\infty$ means that the value of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a



$\lim_{x\to 0} g(x)=+\infty$

② $\lim_{x\to a} f(x)=-\infty$



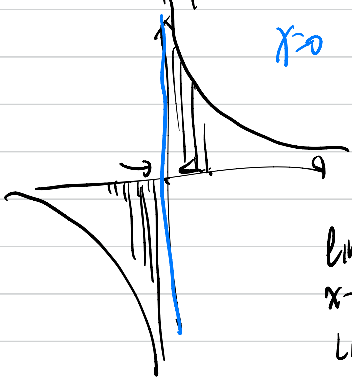
$x=0$ is v.a

$f(x)=-\frac{1}{x^2}$ $x\to 0$

$\lim_{x\to 0} f(x)=-\infty$

③ $\lim_{x\to a^+} f(x)=+\infty$

④ $\lim_{x\to a^-} f(x)=-\infty$



$x=0$ is v.a

$f(x)=\frac{1}{x}$

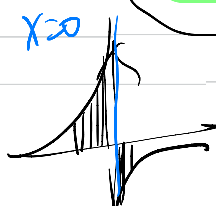
⑤ $\lim_{x\to a^-} f(x)=+\infty$

⑥ $\lim_{x\to a^+} f(x)=-\infty$

$\lim_{x\to 0^+} f(x)=+\infty$

$\lim_{x\to 0^-} f(x)=-\infty$

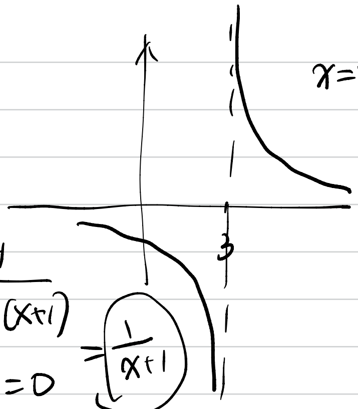
$x=0$



$f(x)=\frac{-1}{x}$

$x=a$ is vertical asymptote

$f(x) = \frac{1}{x-3}$



$x=3$ is V.A

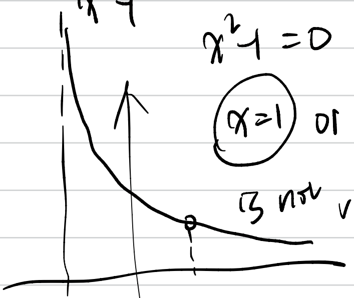
$f(x) = \frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \boxed{\frac{1}{x+1}}$

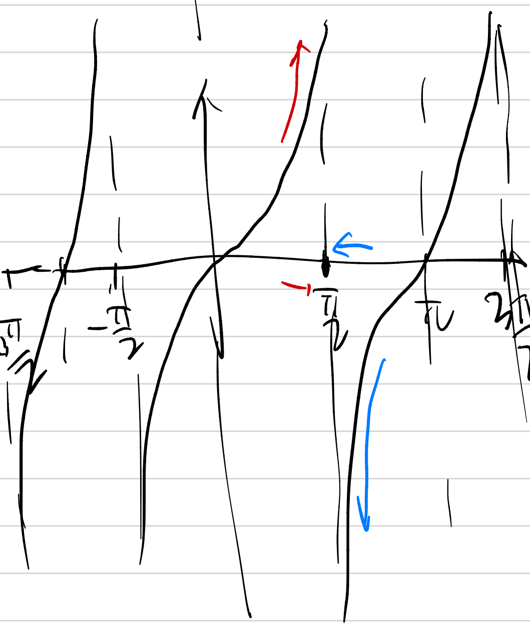
$x^2-1=0$

$x=1$ or $x=-1$ is V.A

$x=-1$

is not V.A





$x = -\frac{3\pi}{2}$ is V.A

$x = \frac{\pi}{2}$ is V.A

$x = \frac{3\pi}{2}$ is V.A

$$\lim_{x \to \frac{\pi}{2}^-} \tan x = +\infty$$

$x = \pm \frac{(2k+1)\pi}{2}$ is V.A

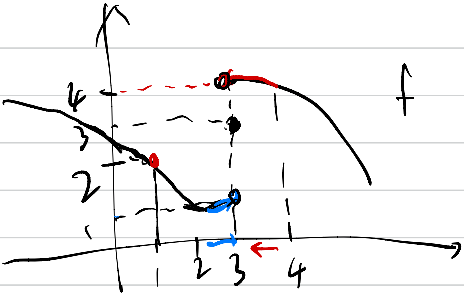
$$\lim_{x \to \frac{\pi}{2}^+} \tan x = -\infty$$

$\sin\left(\frac{2k+1}{2}\pi\right) = \pm 1$

$\tan x = \frac{\sin x}{\boxed{\cos x}}$

$\cos x = 0$ iff $x = \frac{(2k+1)\pi}{2}$

is V.A



a) $\lim_{x\to 1} f(x) = 2$

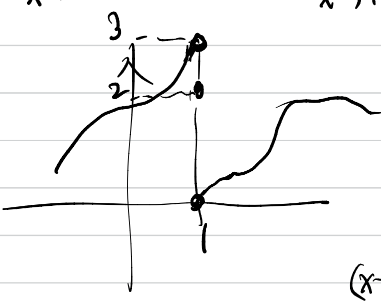
$\lim_{x\to 3^-} f(x) = 1$

$\lim_{x\to 3^+} f(x) = 4$

$\lim_{x\to 3} f(x)$ doesn't exist

$f(3) = 3$

$\lim_{x\to 1^-} f(x)=3$ $\lim_{x\to 1^+} f(x)=0$ $f(1)=2$



$f(x)=\frac{1}{(x-5)^3}$

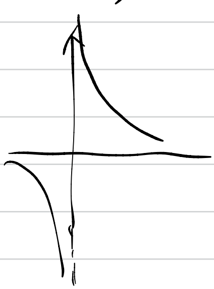
$(x-5)^3>0$

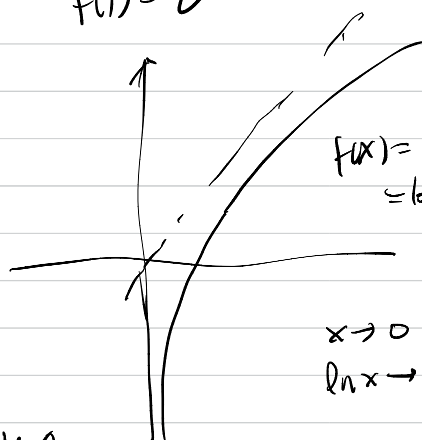
$x\to 5^+$ $f(x)\to +\infty$

We have $x=5$ to be V.A

$x\to 5^-$ $x-5<0$ $(x-5)^3<0$

$f(x)\to -\infty$





$f(x)=\ln x$
$=\log_e x$

$x\to 0$
$\ln x\to -\infty$

$\lim_{x\to 0^+} \ln x = -\infty$

0 is V.A

$g(x) = \dfrac{x^2-2x-8}{x-4}$ ← $x=4$ denominator $=0$

$= \dfrac{(x-4)\ (x+2)}{x-4}$ x^2-2x-8 when $x=4$

$16-2\times4-8=16-8-8=0$

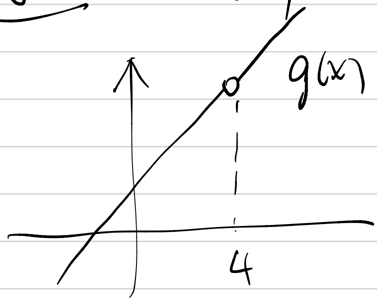
$x^2-2x-8 = \underline{(x-4)(x+2)}$

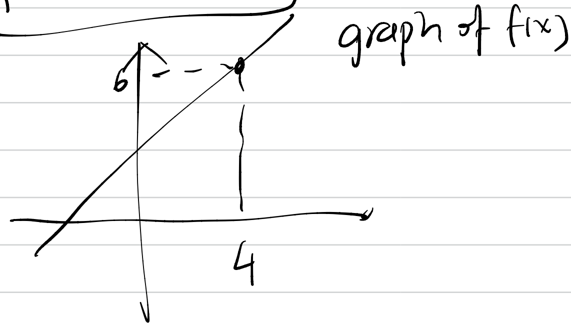
$g(x) = \dfrac{(x-4)(x+2)}{(x-4)}$ consider every point that $\neq 4$ but close to 4

$\lim\limits_{x\to4} g(x) = \lim\limits_{x\to4} (x+2) = 6$

$\underline{g}(x) = \dfrac{(x-4)(x+2)}{x-4}$

$\boxed{f(x) = x+2}$





$\lim\limits_{x\to4} g(x) = \lim\limits_{x\to4} f(x)$

Limit laws

c is constant

$\lim_{x\to a} f(x)$ $\lim_{x\to a} g(x)$ exist!!

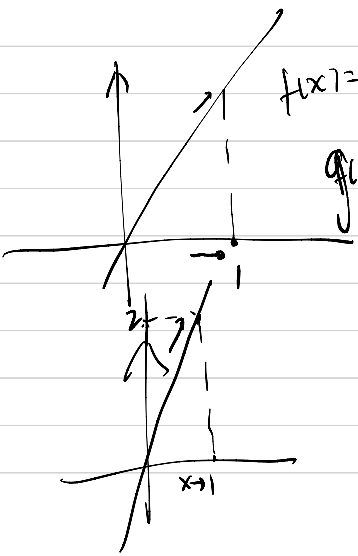
① $\lim_{x\to a} [f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

② $\lim_{x\to a} (f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

③ $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$

④ $\lim_{x\to a} (f(x)\,g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

⑤ $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$



$f(x) = x$

$g(x) = 2x$
$= 2f(x)$

$\lim_{x\to 1} f(x) = 1$

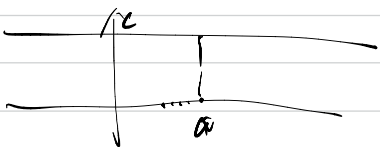
$\lim_{x\to 1} g(x) = \lim_{x\to 1} 2\cdot f(x) = 2 \lim_{x\to 1} f(x)$

$= 2\cdot 1 = 2$

6. $\lim_{x\to a} (f(x))^n = \left(\lim_{x\to a} f(x)\right)^n$ n is positive integer

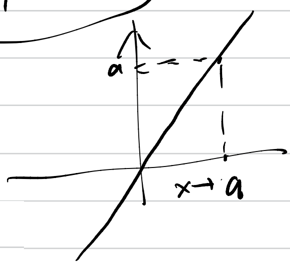
7 $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ n is even, we need to make sure $\lim_{x\to a} f(x) \geq 0$

8 $\lim_{x\to a} c = c$



9. $\lim_{x\to a} x = a$

$y = x$



10 $\lim_{x\to a} x^n = a^n$

$\lim_{x\to a} x^n$

$= \left(\lim_{x\to a} x\right)^n$

$= a^n$

11 $\lim_{x\to a} \sqrt[n]{x} = \lim \sqrt[n]{a}$

Example ①

$f(x)$ $5 \in$ Domain of $f(x)$

$\lim_{x\to 5} (2x^2 - 3x + 4) = \lim_{x\to 5} 2x^2 - \lim_{x\to 5} 3x + \lim_{x\to 5} 4$

$= 2\lim_{x\to 5} x^2 - 3\lim_{x\to 5} x + 4$

$= 2\left(\lim_{x\to 5} x\right)^2 - 3 \cdot 5 + 4$

$= 2 \cdot 5^2 - 15 + 4$

$= 2 \cdot 25 - 15 + 4 = 39$

$\lim_{x\to 5} f(x) = f(5)$

$= 2 \cdot 5^2 - 3 \times 5 + 4$

$= 39$

$\dfrac{\text{polynomial}}{\text{polynomial}}$

f is a polynomial or a rational function $a \in$ Domain of f

$\lim_{x \to a} f(x) = f(a)$

b) $\lim_{x \to -2} \dfrac{x^3+2x^2-1}{5-3x} = \dfrac{\boxed{\lim_{x\to -2} x^3+2x^2-1}}{\lim_{x\to -2} 5-3x} = \dfrac{\lim_{x\to -2} x^3 + \lim_{x\to -2} 2x^2 - \lim_{x\to -2} 1}{\lim_{x\to -2} 5 - \lim_{x\to -2} 3x}$

$= \dfrac{(\lim_{x\to -2} x)^3 + 2\lim_{x\to -2} x^2 - 1}{5 - 3\lim_{x\to -2} x}$

$-2 \quad \boxed{\dfrac{x^3+2x^2-1}{5-3x}}$

$5-3x=0$

$\Leftrightarrow x = \dfrac{5}{3}$

$= \dfrac{(-2)^3+2(-2)^2-1}{5-3(-2)}$

Domain $= \mathbb{R} - \{\frac{5}{3}\}$

$-2 \in$ Domain

$= \dfrac{-8+2\times(4)-1}{5-(-6)} = \dfrac{-1}{11}$

$\lim_{x \to -2} f(x) = f(-2) = \dfrac{\cancel{-8+8}-1}{5-3(-2)} = \dfrac{-1}{11}$