$[6,9]$

Class 3.

Domain

① Domain of the function $\boxed{f(x)=\sqrt{54+3x-x^2}}$

let $54+3x-x^2 \geq 0$

$\Rightarrow x^2 \boxed{-3}x-54 \leq 0$

factorize

$\boxed{(x-9)(x+6) \leq 0}$ $\Rightarrow$ $x \in [-6, 9]$ square bracket

$(x-a)(x-b) \leq 0$ x bounded by a and b

$-9+6=\boxed{-3}$

$-54=\boxed{-9\times 6}$

$=-6\times 9$

① $(x-a) \geq 0$, $(x-b) \leq 0$

$(x-a)(x-b) \leq 0$

② $(x-a) \leq 0$, $(x-b) \geq 0$

② $f(x)=x^3+8x^2$ $g(x)=7x^2-3$ $\frac{f}{g}$ undefined

$\frac{f}{g}=\frac{x^3+8x^2}{7x^2-3}$ when $g(x)=0$ $\frac{f}{g}$ is Not defined

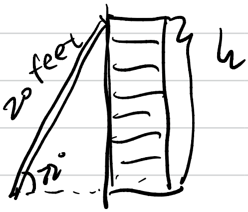
$7x^2-3=0$ $x^2=\frac{3}{7}$

$x=\sqrt{\frac{3}{7}}$ or $x=-\sqrt{\frac{3}{7}}$

③ $\frac{4}{17}\pi =$ ____ degree

$4^\circ =$ ____ radians

④ 20 feet ladder leans against building, angle between ground and ladder is 72 degree
How height is building?



$\frac{h}{20}=\sin 72^\circ$

$h=\sin 72^\circ \cdot 20$ feet

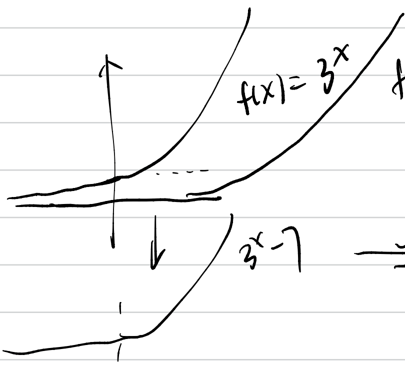
$=\sin\left(\frac{72}{180}\cdot\pi\right)\cdot 20$ feet

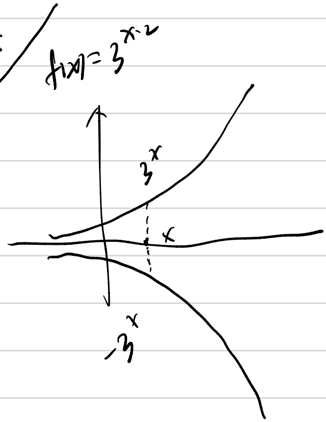
⑤ $f(x)=3^x$ write equation of the graph that results from

a) shifting two 7 units downward $y=3^x-7$

b) 2 units to right $y=3^{x-2}$

c) reflecting about x-axis $y=-3^x$





exponential function $f(x)=b^x$, b positive constant

when x is positive integer n

$$f(n)=b^n=\underbrace{b\times b\times b\times\cdots b}_{n}$$

when $x=-n$

$$f(-n)=b^{-n}=\frac{1}{b^n}$$

when x is rational number $=\frac{m}{n}$

$$f(x)=f\left(\frac{m}{n}\right)=b^{\frac{m}{n}}=\left(\sqrt[n]{b}\right)^m$$

what if x is irrational number

$b^{\sqrt{2}}$ $\quad \sqrt{2}=1.414\cdots$

$1\le\sqrt{2}\le2$

$1.4\le\sqrt{2}\le1.5$

$1.41\le\sqrt{2}\le1.42$

$1.414\le\sqrt{2}\le1.415$

given b

$f(1)=b$

$f(1.4)=b^{1.4}$

$f(1.41)=b^{1.41}$

$f(1.414)=b^{1.414}$

have definition

increasing

$b^{\sqrt{2}}$

$f(2)=b^2$

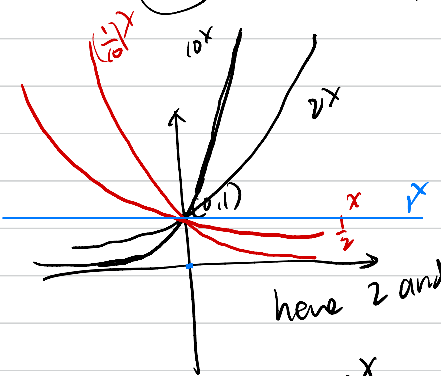
$f(1.5)=b^{1.5}$

$f(1.42)=b^{1.42}$

$f(1.415)=b^{1.415}$

decreasing

converge to unique value



here 2 and 10 are both >1

$f(x)=2^x$

$f(0)=2^0=1$

$f(x)=10^x$

Domain $\mathbb{R}$

Range: $(0,+\infty)$

Law of exponent

a and b positive number, x, y are real number

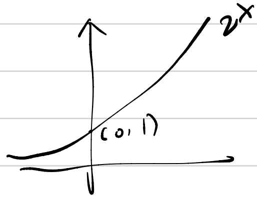
① $b^{x+y} = b^x \cdot b^y$

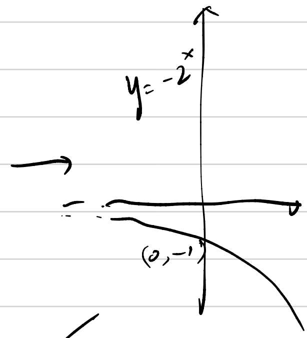
② $b^{x-y} = \frac{b^x}{b^y} = b^x \cdot b^{-y}$ ← $\frac{1}{b^y}$

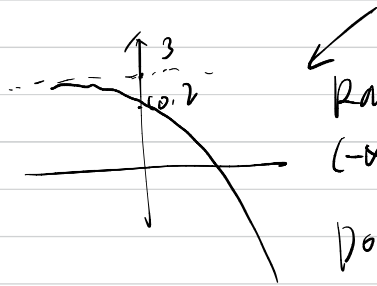
③ $(b^x)^y = b^{xy}$

④ $(ab)^x = a^x \cdot b^x$

$y = 3 - 2^x$





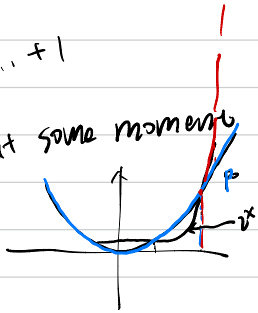


Range
$(-\infty, 3)$

Domain $\mathbb{R}$

$p(x) = 10x^{10} + 9x^9 + \dots + 1$

$f(x) = 2^x$ at some moment

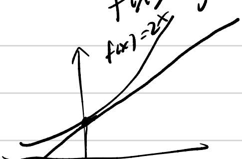


e

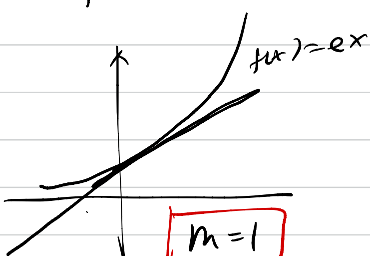
$f(x) = 2^x$

$f(x) = 3^x$

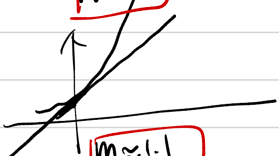
$f(x) = e^x$



$\boxed{m \approx 0.7}$



$\boxed{m = 1}$



$\boxed{m \approx 1.1}$

$0.7 \le 1 \le 1.1$

$2 \le e \le 3$

$e \approx 2.71828\dots$

$e^{2x} - 4e^x + 3 = 0$

$3 = -1x - 3$

$(-1) + (-3) = -4$

$a^2 - 4a + 3 = 0$

$e^x = a$

$(a-1)(a-3) = 0$

$a = 1$ or $= a = 3$

$\boxed{e^x = 1}$ or $\boxed{e^x = 3}$

$x = \ln 3$

$f(x) = b^x$

$b > 0$

$x = 0$ $e^0 = 1$

$e^x = b$ ← constant

$x = \ln b$

Logarithm $b > 0$

$f(x) = \log_b x := y$

$b^y = x$

$\boxed{\log_e b} = x$

$\log_e = \ln$

$e^x = b$

$e^x = b$

$\log_2 16 = y$ $\quad 2^y = 16 \quad 2^4 = 16$

$y = 4$

$\log_2 16 = 4$

$3^2 = 9$

$\log_2 16 = 4$

$\to 2^? = 16$

$\log_3 9 = 2$

$3^? = 9$

$\log_5 \sqrt{125}$

$= \log_5 5^{\frac{3}{2}} = \frac{3}{2}$

$5^? = 5^{\frac{3}{2}}$

$125 = 5^3 \quad \sqrt{125} = (5^3)^{\frac{1}{2}} = 5^{\frac{3}{2}}$

$$\boxed{b^{\log_b x} = x}$$

$$\boxed{\log_b (b^x) = x}$$

$\ln x = \log_e x$

$\log_b x = ?$

$b^? = x$

$e^{5-3x} = 10$

$2^{x-5} = 2^2$

$x - 5 = 2$

$x = 7$

$\log_2 (2^{x-5}) = \log_2 (2^2)$

$x - 5 = 2$

$\ln (e^{5-3x}) = \ln(10)$

$\ln = \log_e$

$5 - 3x = \ln(10)$

$3x = 5 - \ln(10)$

$x = \frac{5 - \ln(10)}{3}$

$$b^{\log_b(x)+\log_b(y)}$$

law of Logarithm

① $\log_b(x\cdot y) = \log_b x + \log_b y$

② $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

③ $\log_b(x^n) = n\log_b x$

$\log_b(xy) = a$ //

$b^{\boxed{\log_b(xy)}} = xy = b^{\log_b(x)} \cdot b^{\log_b(y)}$

$b^{\log_b(x)} = x \qquad b^{\log_b(y)} = y$

$\frac{x}{y} = x\cdot y^{-1} \qquad \log_b(x\cdot y^{-1}) = \log_b(x) + \log_b(y^{-1})$

$= \log_b(x) - \log_b(y)$

$\log_2 80 - \log_2 5 \qquad 80 = 2^4 \cdot 5 = 16\times 5$

$= \log_2(2^4 \times 5) - \log_2 5$

$= \log_2(2^4) + \cancel{\log_2(5) - \log_2 5}$

$= \log_2(2^4) = 4$

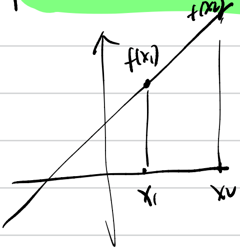
$$\log_b a = \frac{\ln a}{\ln b}$$

$\ln(x^2\cdot\sqrt{x^2+1}) = \ln(x^2) + \ln(\sqrt{x^2+1})$

$= 2\ln(x) + \ln\left((x^2+1)^{\frac{1}{2}}\right)$

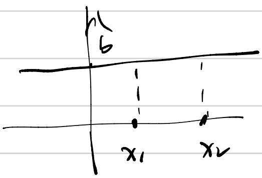
$= 2\ln(x) + \frac{1}{2}\ln(x^2+1)$

f. called 1-1 if it never take same value $f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$

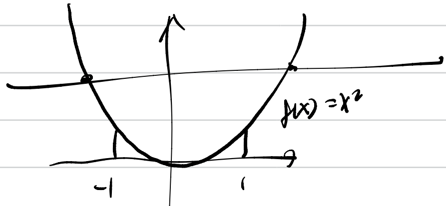


$f(x) = mx+b$

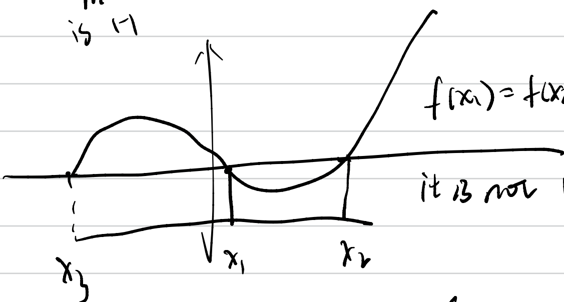
$m \neq 0$
is 1-1



$f(x) \equiv b$ constant function
is not 1-1

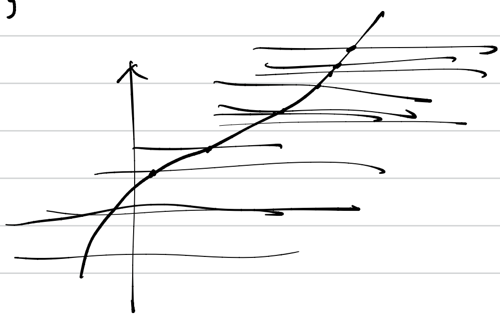


$f(1) = 1$
$f(-1) = (-1)^2 = 1$

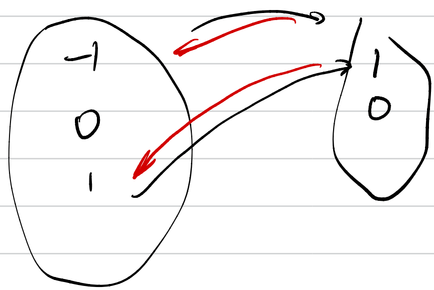


$f(x_1) = f(x_2) = f(x_3)$

it is not 1-1

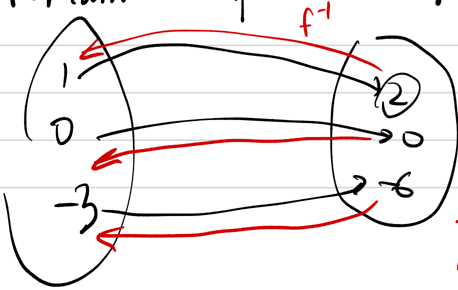


$f = x^2$ $\quad f^{-1}$



Domain $\quad f \quad$ Range $\quad f(x) = 2x$

$f^{-1}(x) = \frac{x}{2}$



$f^{-1}(y) = x$
s.t. $f(x) = y$
$y \in$ Range of f

$f^{-1}(2) = 1$ $\quad f^{-1}(-6) = -3$

$f^{-1}(0) = 0$

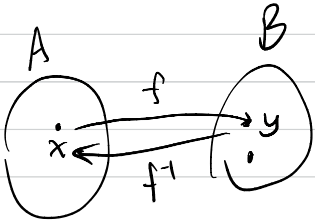
Let f be 1-1 functions domain A and Range B.

f^{-1} inverse function has domain B and Range A

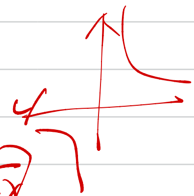
$f^{-1}(y)=x \iff f(x)=y$

$f: x \longmapsto y$

$f^{-1}: y \longmapsto x$



$f^{-1}(x) \neq \cancel{\frac{1}{f(x)}}$



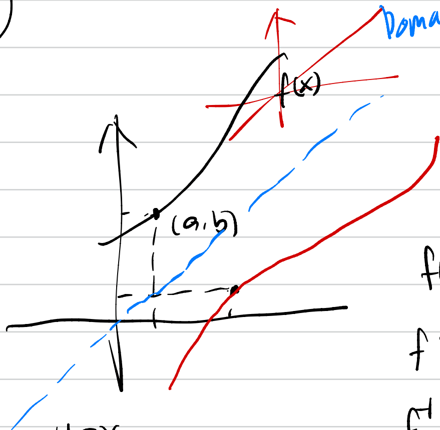
$f(x)=2x$ $\quad \frac{1}{f(x)}=\frac{1}{2x}$

$f^{-1}(x)=\frac{1}{2}x$ $\quad$ different

Domain $\mathbb{R}-\{0\}$

Domain $\mathbb{R}$

$$\begin{cases} f^{-1}(f(x))=x \\ f(f^{-1}(y))=y \end{cases}$$



$f(a)=b$

$f: a \longmapsto b$

$f^{-1}: b \longmapsto a$

$f^{-1}(b)=a$

(b,a) locate at graph of f^{-1}

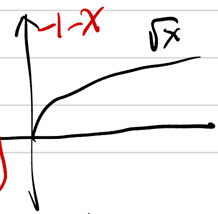
$x \geq 0$

$y=\sqrt{-1-x}=\sqrt{-(x+1)}$ adding 1 after x coordinate

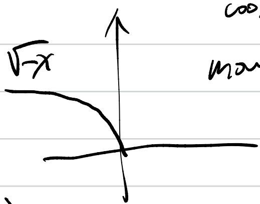
$y^2=-1-x$

$\Downarrow$

$\boxed{x=-1-y^2}$

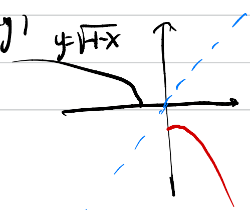


$\Rightarrow$

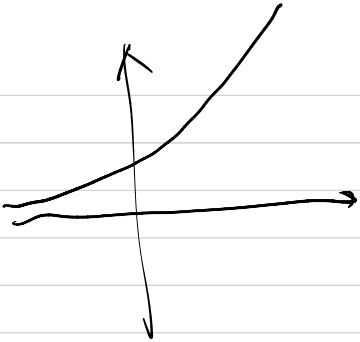


moving left by 1

$\Rightarrow$



$f^{-1}(y)=-1-y^2 \rightarrow f^{-1}(x)=\underline{-1-x^2}$



$f(x)=b^x$ $\quad f^{-1}(x)=\log_b x$

$x \mapsto b^x = y \quad x \leftarrow b^x$

$f(x)=b^x$ it is a 1-1 function

$f^{-1}(y)=\log_b y = x$ x we are looking for is the number satisfy

$$\boxed{b^x=y}$$

$\log_e(a\cdot b) = \ln(a)+\ln(b)$

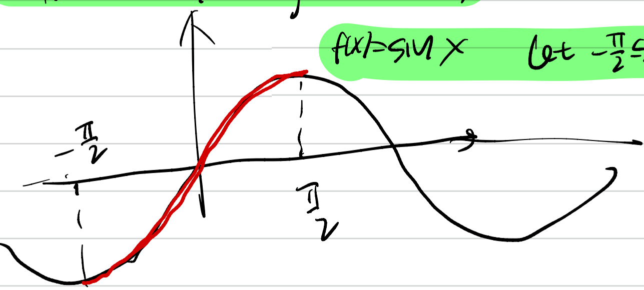
ln

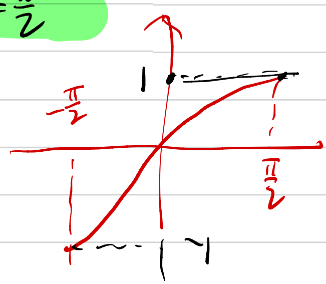
$\ln(a)+\frac{1}{2}\ln(b) = \ln(a)+\ln(\sqrt{b})$

$= \ln(a\cdot\sqrt{b})$

inverse of trig function

$f(x)=\sin x$ Let $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$





$f(x)=\sin x \quad x\in[-\frac{\pi}{2},\frac{\pi}{2}]$

at this moment $f^{-1}(x)$ exist $\sin^{-1}(x)$

$=\arcsin(x)$

$\arcsin(1)=\frac{\pi}{2}$ angle satisfy

$\arcsin(\frac{1}{2})=\theta$

$\sin(\theta)=\frac{1}{2}$ $\quad \sin(\frac{\pi}{6})=\frac{1}{2}$

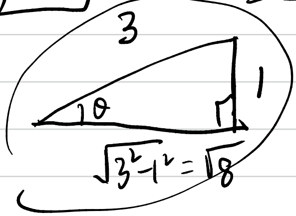
$\boxed{\theta=\frac{\pi}{6}}$ $\quad \cancel{\frac{\pi}{6}+2\pi \text{ will work?}}$

graph for arcsin $\sin^{-1}$

$\boxed{\arcsin\left(\frac{1}{3}\right) = \theta}$ s.t $\sin(\theta) = \frac{1}{3}$

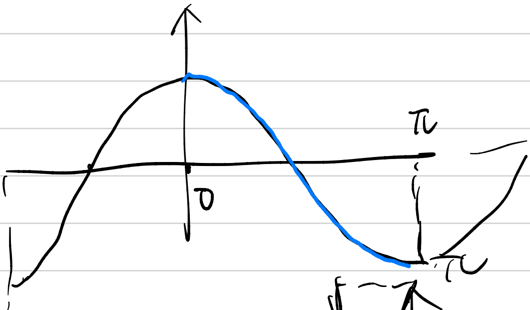
$\cos\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{\sqrt{8}}{3}$

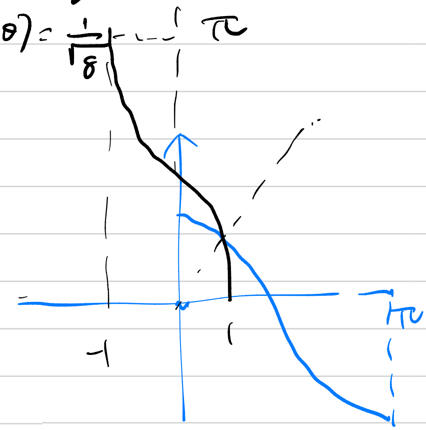
$\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{\sqrt{8}}$

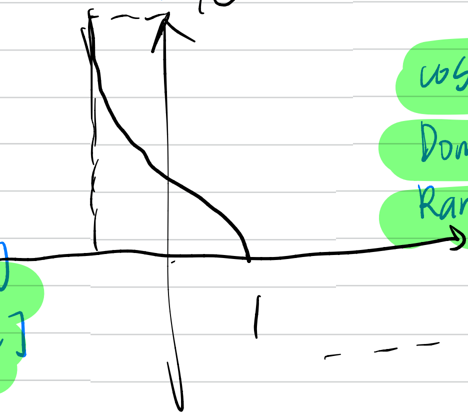


$\cos(\theta) = \frac{\sqrt{8}}{3}$

$\tan(\theta) = \frac{1}{\sqrt{8}}$

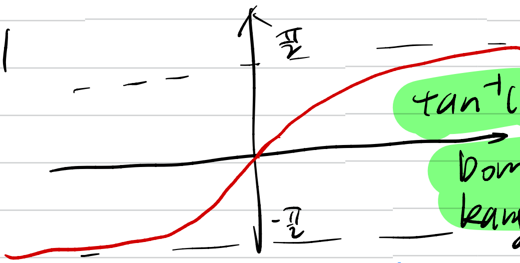




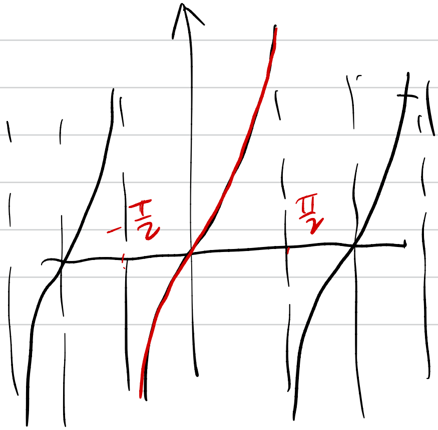


for $\arccos(x)$
Domain $[-1, 1]$
Range $[0, \pi]$

$\cos x$ on $[0, \pi]$
Domain
Range $= [-1, 1]$



$\tan^{-1}(x)$
Domain $\mathbb{R}$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



end point is not included

$\tan(x)$:
Domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Range $\mathbb{R}$

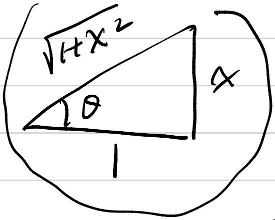
$\tan\left(\frac{\pi}{2}\right) = \frac{1}{0}$ not define
same for $-\frac{\pi}{2}$

e.g Simply the express $\boxed{\cos(\tan^{-1}x)}$

$\tan^{-1}x = \theta$

$\Downarrow$

$\tan\theta = x$



$\cos(\tan^{-1}x)$

$= \cos\theta$

$= \dfrac{1}{\sqrt{1+x^2}}$

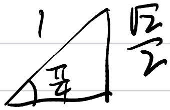
$\sin\left(\underbrace{\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)}\right)$

$\uparrow$

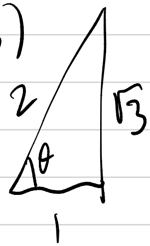
$\theta \in [0, \pi]$

$\boxed{\cos\theta = \frac{\sqrt{2}}{2}} = \frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$



$\tan^{-1}(\sqrt{3}) = \theta = \frac{\pi}{3}$

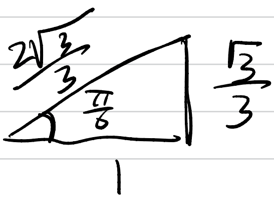


$\frac{\pi}{3}$

$\tan(\theta) = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$



$\sqrt{1+\frac{3}{9}} = \sqrt{\frac{12}{9}}$

$= \frac{2\sqrt{3}}{3}$