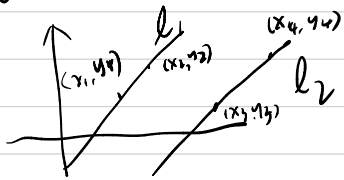
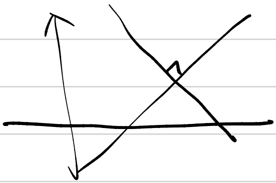
class 2.



$l_1 \parallel l_2$ $\quad m_1 = m_2$

$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \frac{y_4 - y_3}{x_4 - x_3}$



$m_1 \cdot m_2 = -1$

l_1 $\boxed{4x+4y=3}$ $\quad l_2$: cross $(9, 8)$

$4x+4y=3$

$y = mx + b$

$4y = 3 - 4x$ $\quad m$ - slope

$\boxed{y = \frac{-4x}{4} + \frac{3}{4} = -x + \frac{3}{4}}$

slope $= -1$ $\quad$ for l_2, slope -1

$(9, 8)$

$y = -x + b$

$8 = -9 + b$ $\quad b = 17$ $\quad y = -x + 17$

l_1 $\boxed{2x+3y=4}$ $\quad$ find a line perpendicular to l_1 $\quad \boxed{(3, 8)}$

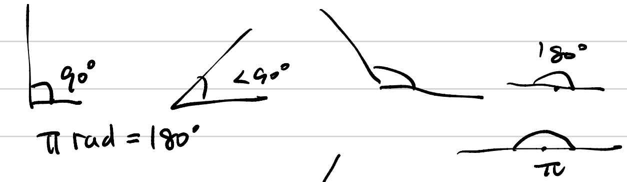
$2x+3y=4 \Rightarrow 3y = 4 - 2x$

$y = \frac{4-2x}{3} = \frac{-2x}{3} + \frac{4}{3}$

$m_1 = \frac{-2}{3}$ $\quad m_2 = \frac{3}{2}$ $\quad y = \frac{3}{2}x + b$

$8 = \frac{3}{2} \times 3 + b = \frac{9}{2} + b$ $\quad b = 8 - \frac{9}{2} = \frac{7}{2}$

$y = \frac{3}{2}x + \frac{7}{2}$

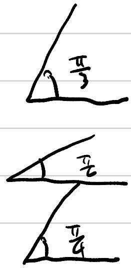


$\pi \text{ rad} = 180°$

$\frac{\pi}{3} \text{ rad} = 60°$

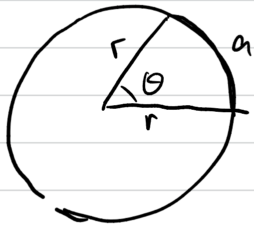
$\frac{\pi}{6} \text{ rad} = 30°$

$\frac{\pi}{4} \text{ rad} = 45°$



$35° = \frac{35 \cdot \pi \text{ rad}}{180} = \frac{7\pi}{36} \text{ rad}$

$\frac{1\pi \text{ rad}}{36} = \frac{\pi \text{ rad}}{36} \times \frac{180}{\pi \text{ rad}} = 5°$

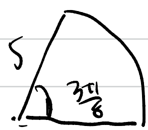


$\frac{a}{2\pi r} = \frac{\theta}{360}$

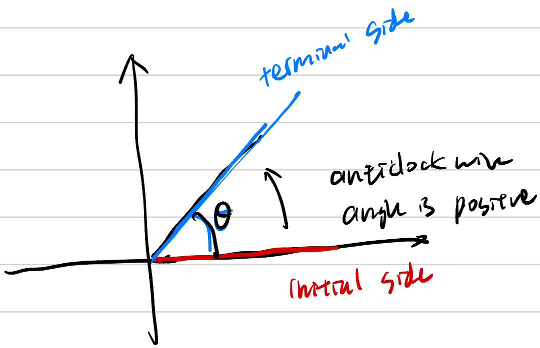
$\frac{a}{r} = \theta$

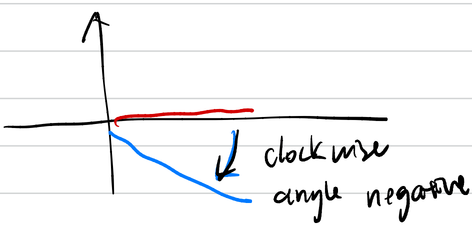
$\boxed{a = \theta \cdot r}$

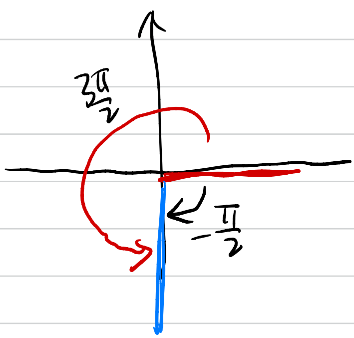
$\theta = \frac{a}{r} = \frac{5}{6} \text{ rad.}$

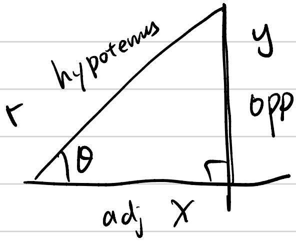


$a = \frac{3\pi}{8} \cdot 5 = \frac{15\pi}{8}$





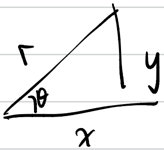


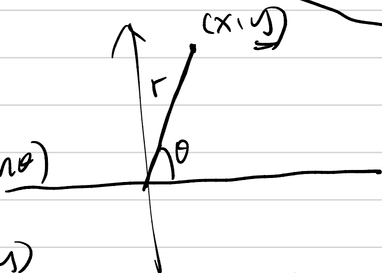


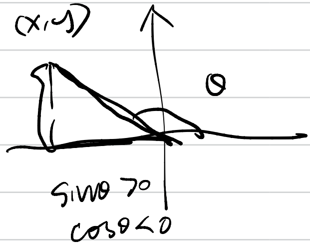
$$\sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x} \qquad \cot = \frac{x}{y}$$

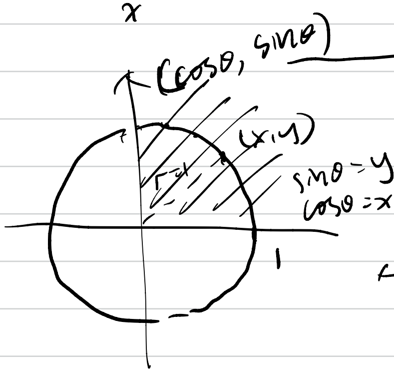
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

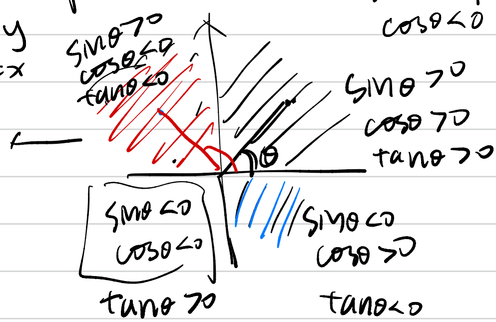
$$\csc\theta = \frac{r}{y}$$

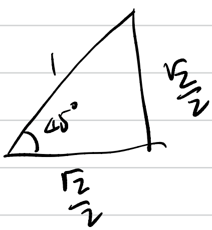


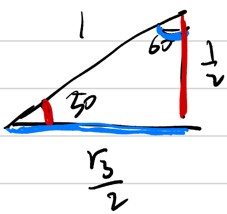


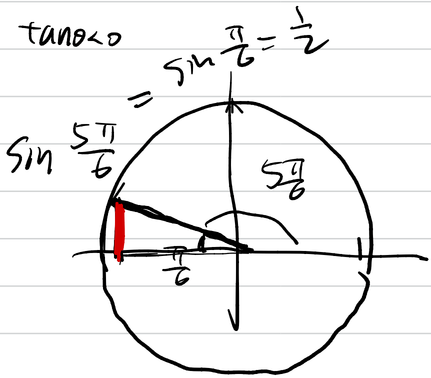










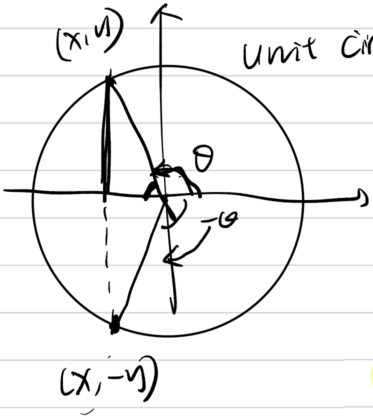


$45° = \frac{\pi}{4}$ $\qquad 30° = \frac{\pi}{6}$ $\qquad 60° = \frac{\pi}{3}$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

unit circle



$\sin\theta = \sin(\pi-\theta)$

$\sin\theta = y$

$\sin(\pi-\theta) = y$

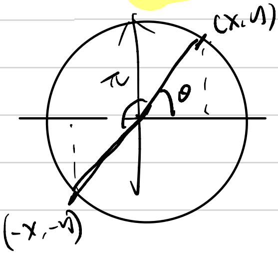
$\cos\theta = x = x = \cos(-\theta)$

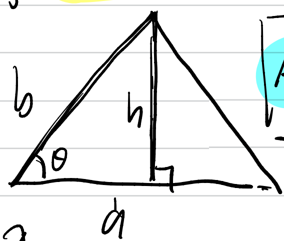
$\sin(-\theta) = -y = -\sin\theta$

$\sin(-\theta) = -\sin\theta$

$\cos\theta = \cos(-\theta)$

$\sin(\pi+\theta) = -y = -\sin\theta$

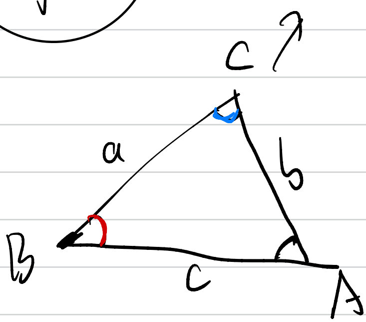




$\boxed{A = \frac{ah}{2} = \frac{a\cdot\sin\theta\cdot b}{2} = \frac{1}{2}ab\sin\theta}$

$\frac{h}{b} = \sin\theta$

$h = \sin\theta\cdot b$



$\frac{1}{2}b\cdot c\cdot\sin A = \frac{1}{2}a\cdot c\cdot\sin B$

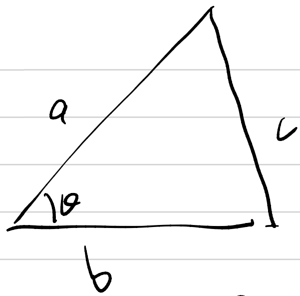
$= \frac{1}{2}a\cdot b\cdot\sin C$

$\frac{bc\cdot\sin A}{abc} = \frac{ac\sin B}{abc} = \frac{ab\sin C}{abc}$

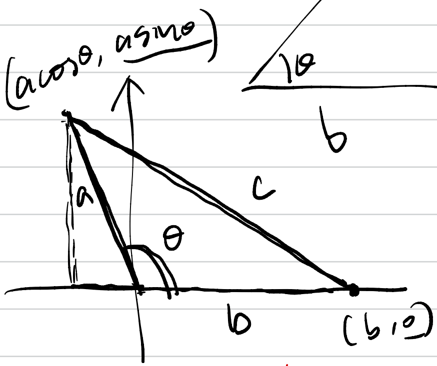
law of sine $\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

law of cos



$$c^2 = a^2 + b^2 - 2ab\cos c$$



$$c = \sqrt{(a\sin\theta - 0)^2 + (a\cos\theta - b)^2}$$

$$c^2 = a^2\sin^2\theta + (a\cos\theta - b)^2$$

$$= a^2\sin^2\theta + a^2\cos^2\theta - 2ab\cos\theta + b^2$$

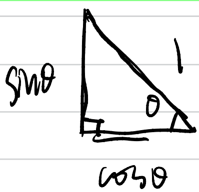
prove by using distance betwee two points

$$= a^2(\sin^2\theta + \cos^2\theta) - 2ab\cos\theta + b^2$$

$$= a^2 - 2ab\cos\theta + b^2$$

$$= a^2 + b^2 - 2ab\cos\theta$$

① $\cos^2\theta + \sin^2\theta = 1$



$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\sin(x-y) = \sin x\underbrace{\cos(-y)}_{\cos y} + \cos x\underbrace{\sin(-y)}_{-\sin y}$$

$$= \sin x\cos y - \cos x\sin y$$

$$\cos(x-y) = \cos x\cos(-y) - \sin x\sin(-y)$$

$$= \cos x\cos y + \sin x\sin y$$

$y = x$

$$\sin(x+x) = \sin(2x) = \sin x\cos x + \cos x\sin x$$

$$= 2\sin x\cos x$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos(x+x) = \cos(2x) = \cos x\cos x - \sin x\sin x = \boxed{\cos^2 x - \sin^2 x}$$

①

$$= (1-\sin^2 x) - \sin^2 x$$

② $\boxed{= 1 - 2\sin^2 x}$ ③

$$= \boxed{\cos^2 x - (1-\cos^2 x)}$$

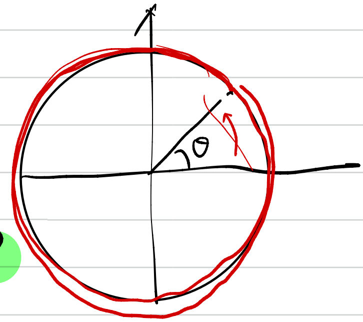
$$= 2\cos^2 x - 1$$

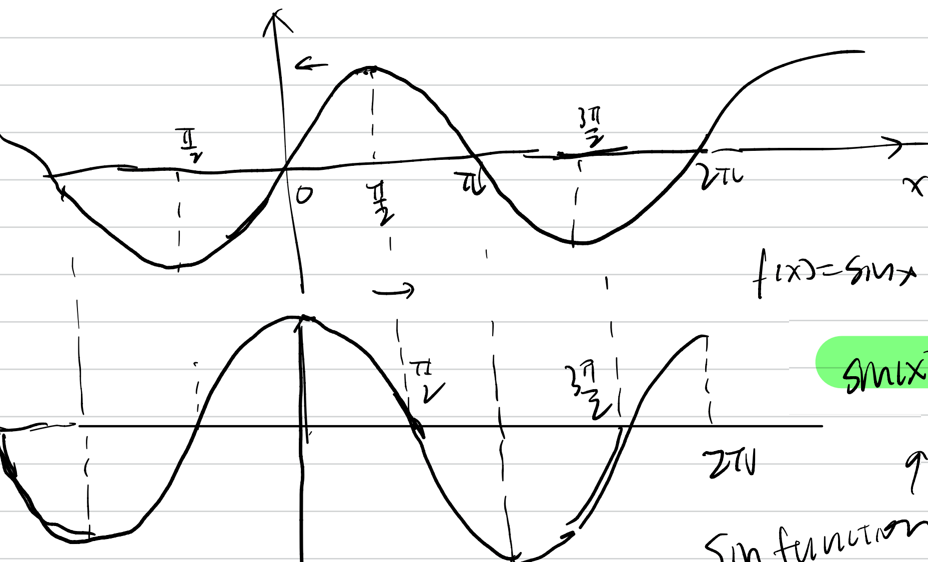
$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x\cos y + \cos x \sin y}{\cos x\cos y - \sin x \sin y}$$

$$= \frac{\underbrace{\boxed{\frac{\sin x}{\cos x}}}_{\tan x}\frac{\cancel{\cos y}}{\cancel{\cos y}} + \frac{\cancel{\cos x}}{\cancel{\cos x}}\underbrace{\boxed{\frac{\sin y}{\cos y}}}_{\tan y}}{\underbrace{\frac{\cos x \cos y}{\cos x \cos y}}_{1=} - \boxed{\frac{\sin x}{\cos x}}\boxed{\frac{\sin y}{\cos y}}} \quad = \tan x \tan y$$

$$= \frac{\tan x + \tan y}{1 + \tan x \tan y}$$

$$\tan(x+x) = \frac{\tan x + \tan x}{1 + \tan x \tan x} = \frac{2\tan x}{1 + (\tan x)^2} = \tan(2x)$$





$f(x) = \sin x$

$\sin(x) = \cos(x - \frac{\pi}{2})$

$= \cos(\frac{\pi}{2} - x)$

Sin function is cos function shift to right by $\frac{\pi}{2}$

cos is sin shift to left

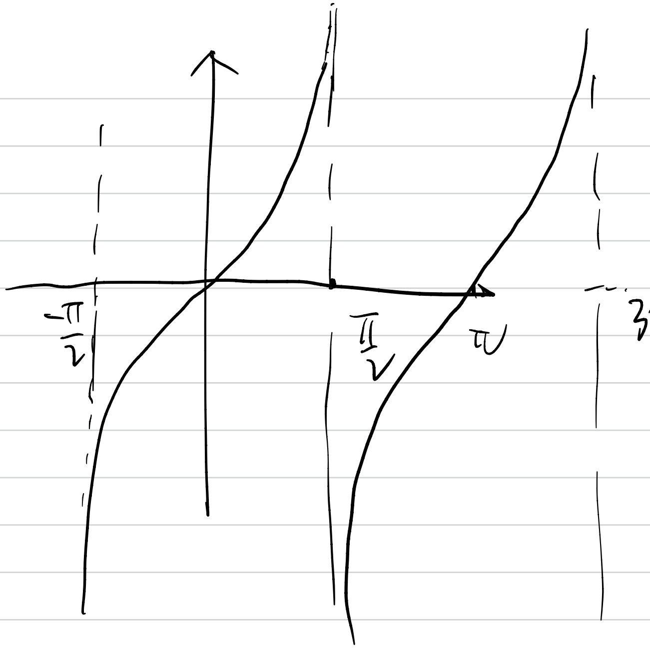
$\cos x = \sin(x + \frac{\pi}{2})$

$= \sin(\pi - (x + \frac{\pi}{2}))$

$= \sin(\frac{\pi}{2} - x)$

f(x) shift to right by unit a

new function $h(x) = f(x-a)$



$$\tan\frac{\pi}{2} = \frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}$$

$$= \frac{\sin\frac{\pi}{2}}{0}$$

Not defined

find all $x \in [0,\pi)$ s.t. $\cos 3x = \frac{1}{2}$

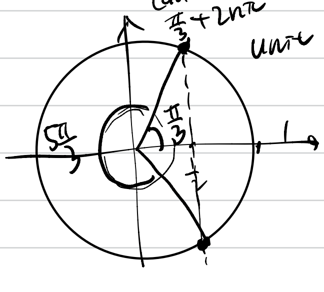
$\cos\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{3}$ $\quad \theta = 2\pi - \frac{\pi}{3}$

$\cos\theta =$

$= \frac{5\pi}{3}$

can be $\frac{\pi}{3} + 2n\pi$

unit



(x,y)

$3x = \theta$

$\boxed{x = \cos\theta}$

$y = \sin\theta$

$x_1 = \frac{\pi}{9}$

$x_2 = \frac{5\pi}{9}$ $\quad x_3 = \frac{7\pi}{9}$

$\frac{5\pi}{3} + 2\pi = \frac{11\pi}{3}$ what about $\frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

$\boxed{x = \frac{11\pi}{9}}$

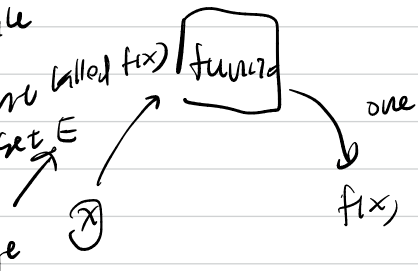
$3x = \frac{7\pi}{3} \ni x = \frac{7\pi}{9}$

$3x = \frac{\pi}{3} + 4\pi \rightarrow x > \pi$

1.1

function f is rule that assign each element x in set D → Domain

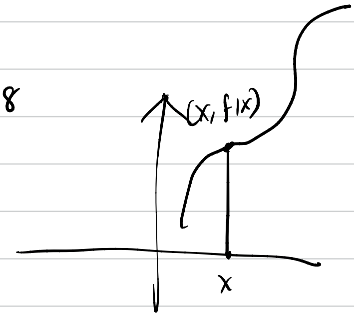
exactly one element called $f(x)$ in set E

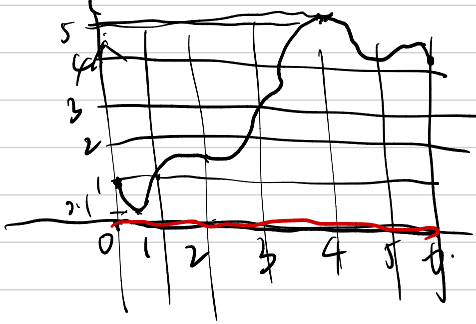


$f(x) = 3x + 8$

graph of function f

$\{(x, f(x)) \mid x \in D\} \subseteq R^2$





Domain is $[0, 6]$

range $[0.1, 5]$

$f(0)=1 \quad f(1)=1 \quad f(3)=3$

Domain of the function is the set of all input for which formula make sense and give us real output

$f(x) = x^2$

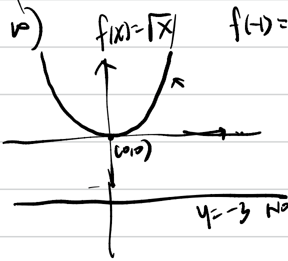
Dom(f) $= (-\infty, \infty)$

Range $= [0, +\infty)$

$f(x) = \sqrt{x}$

$f(-1) = \sqrt{-1} = i$?

$-1 \notin$ Domain of $\sqrt{x}$

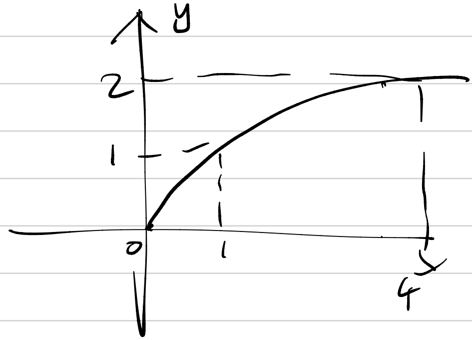


$y=-3$ not intersect with graph

$-3 \notin$ Range(f)

$h(x)=\sqrt{x}$ $\quad$ $\text{Domain}(h)=[0,+\infty)$

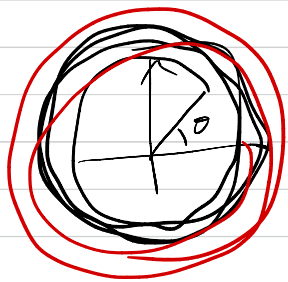
$\text{Range}(h)=[0,+\infty)$

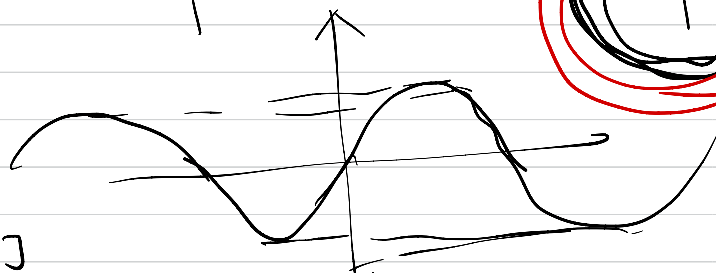


$f(x)=|x|$

$\text{Domain}(f)=(-\infty,\infty)$

$\text{Range}(f)=[0,+\infty)$

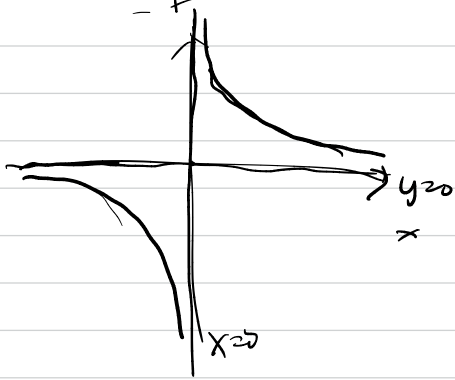




$f(x)=\sin x$

$\text{Dom} f=(-\infty,\infty)$

$\text{Range}(f)=[-1,1]$



$f(x)=\frac{1}{x}$

$\text{Dom} f=(-\infty,\infty)-\{0\}$

$\text{Range} f=(-\infty,\infty)-\{0\}$

$\cancel{f(x)=0}=\frac{1}{x}$ $\quad$ $\cancel{0=1}$

$\mathrm{Dom}(\sqrt{x}) = [0, +\infty)$

$h(x) = \sqrt{x-2}$

$x-2 \geq 0 \iff x \geq 2 \iff \text{Domain } h = [2, \infty)$

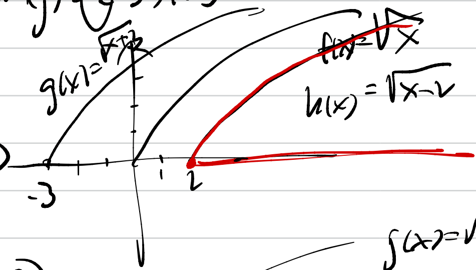
square bracket

$g(x) = \sqrt{x+3}$

$x+3 \geq 0 \quad x \geq -3 \iff \mathrm{Dom}(g) = [-3, +\infty)$

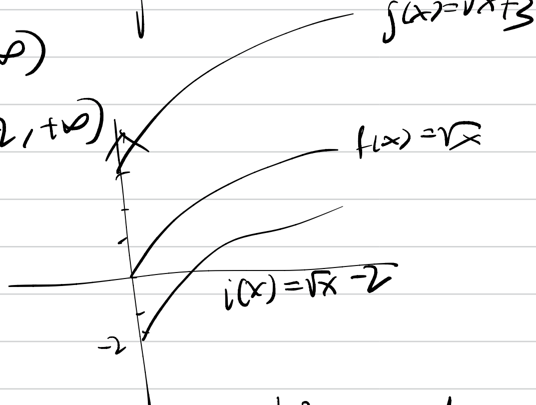
$j(x) = \sqrt{x} + 3 \quad \mathrm{Dom}(j) = [0, +\infty)$

$\mathrm{Range}(j) = [3, +\infty)$



$i(x) = \sqrt{x} - 2 \quad \mathrm{Dom}(i) = [0, +\infty)$

$\mathrm{Range}(i) = [-2, +\infty)$



assume $c > 0$

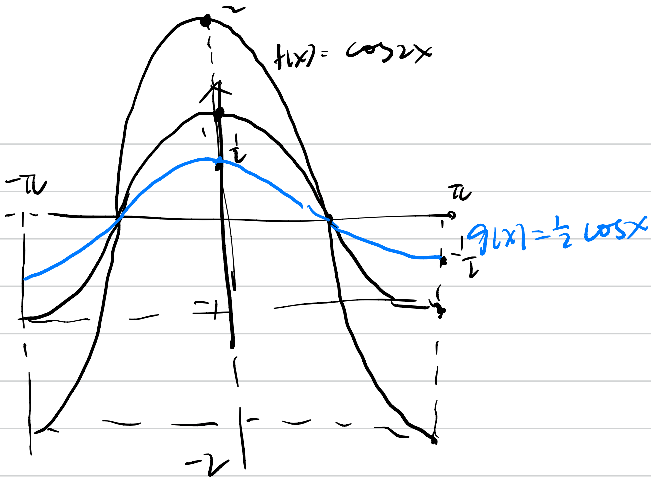
$y = f(x)$ $\quad y' = f(x) + c$ graph of y' is graph of y ↑ by c unit (shift upwards)

$y' = f(x) - c$ downwards by c unit

$y' = f(x-c)$ shifting to right by c unit

$y' = f(x+c)$ shifting to left by c unit

$y = \cos x$



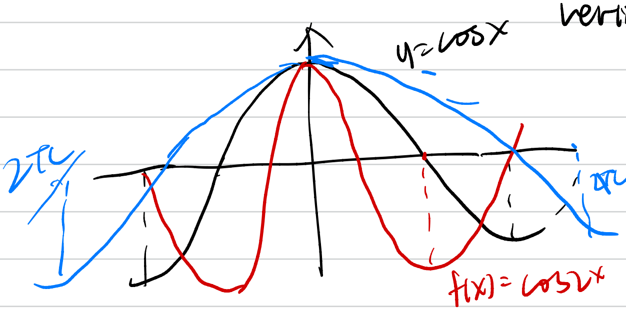
$f(x) = \cos 2x$

$g(x) = \frac{1}{2}\cos x$

$y = f(x)$ $\quad c > 1$

$y = cf(x)$ stretch the graph of $y = f(x)$ vertically by factor c

$y = \frac{1}{c} f(x)$ shrink the graph of $y = f(x)$ vertically by factor of c



$f(x) = \cos 2x$

$x = \frac{\pi}{2}$ $\quad f(\frac{\pi}{2}) = \cos 2 \cdot \frac{\pi}{2}$

$= \cos \pi$

$= -1$

$h(x) = \cos \frac{1}{2}x$

$x = 2\pi$ $\quad h(2\pi) = \cos \frac{2\pi}{2}$

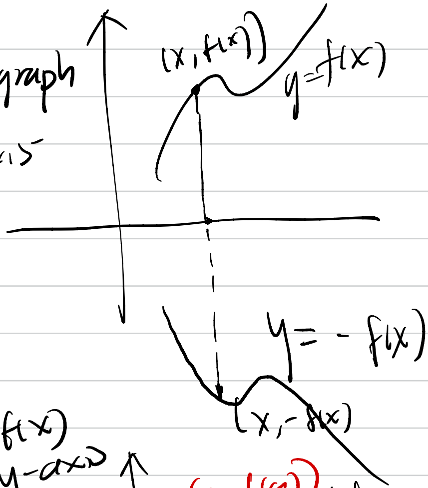
$= \cos \pi$

$= -1$

$y = f(x)$

$y = -f(x)$ reflection the graph about x-axis

$y = f(-x)$



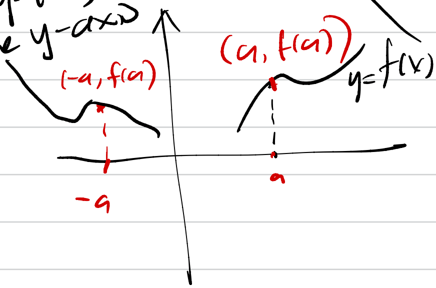
$y = f(-x)$

reflect the graph $y=f(x)$ about the y-axis

$h(x) = f(-x)$

$h(-a) = f(-(-a))$

$= f(a)$



$\text{dom}(h) = \text{dom} f \cap \text{dom} g$

f and g

$$\left\{\begin{array}{l} (f+g)(x) = f(x) + g(x) \\ (f-g)(x) = f(x) - g(x) \\ (fg)(x) = f(x)\,g(x) \end{array}\right.$$

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}} \qquad g(x) \neq 0$$

$\text{dom}(h) = \text{dom} f \cap \text{dom}(g) - \underbrace{\{x \mid g(x) = 0\}}_{?}$

$f(x) = x^2 \quad g(x) = x-1 \qquad \frac{f}{g} = \frac{x^2}{x-1}$

dom f $(-\infty, \infty)$ dom(g) $(-\infty, +\infty)$

$\frac{f}{g}(1) = \cancel{\frac{1}{0}}$

$\text{dom}\left(\frac{f}{g}\right) = (-\infty, \infty) - \{1\}$

$f(x) = \sqrt{x} \qquad f \circ g(x) = f\left(g(x)\right) = \sqrt{x^2+1}$ #

$g(x) = x^2+1 \qquad g \circ f(x) = g(f(x)) = (\sqrt{x})^2+1 = x+1$

given two function f and g.

$f \circ g := f(g(x))$

$f \circ g \neq g \circ f$

e.g. $f(x) = \sqrt{x} \quad g(x) = \sqrt{2-x}$

domain of $f \circ g \qquad f \circ g = f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$

$= \sqrt[4]{2-x}$

$2-x \geq 0$

$x \leq 2$

$\text{Domain}(f \circ g) = \{x \mid x \leq 2\} = (-\infty, 2)$

domain of $g \circ f = g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$

$2-\sqrt{x} \geq 0 \qquad 2 \geq \sqrt{x} \qquad 4 \geq x \geq 0$

Domain of $g \circ f = [0, 4]$