06/06.

Product Rule

$$(fg)' = f \cdot g' + g \cdot f'$$
$$= gf' + fg'$$

$$\frac{d}{dx}(fg)' = f\frac{dg}{dx} + g\frac{df}{dx}$$

$$f(x) = (x^3 - x)e^x$$

$$f'(x) = (x^3 - x)\cdot\frac{d}{dx}e^x + e^x\frac{d}{dx}(x^3 - x)$$
$$= (x^3 - x)e^x + e^x(3x^2 - 1)$$
$$= (x^3 + 3x^2 - x - 1)e^x$$

Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - fg'}{g^2}$$

$f = x \quad g = e^x$

$$h(x) = \frac{x}{e^x}$$

$$h'(x) = \frac{e^x \cdot 1 - xe^x}{(e^x)^2} = \frac{1-x}{e^x} = (-1)\frac{x-1}{e^x}$$

$f(x) = 1 - x \quad g(x) = e^x$

$$h''(x) = \frac{e^x\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}e^x}{(e^x)^2} = \frac{e^x(-1-1+x)}{(e^x)^2} = \frac{x-2}{e^x} = (-1)^2\frac{x-2}{e^x}$$

$$h^{(n)}(x) = (-1)^n \frac{x-n}{e^x}$$

↑ use math induction

$$f(x) = (3x^2-5x)e^x \qquad \frac{d}{dx}x^n = nx^{n-1}$$

$$f'(x) = (3x^2-5x)e^x + e^x(3\times 2\cdot x - 5)$$

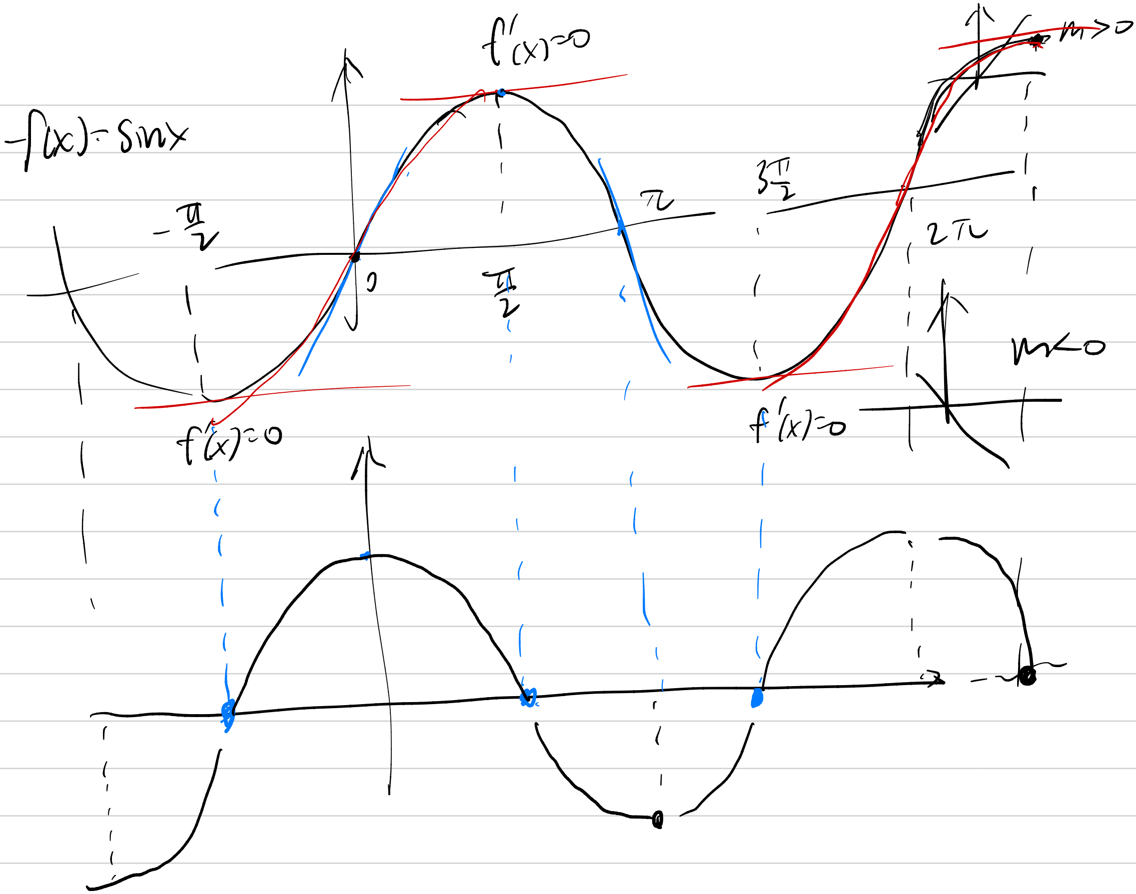
$$= (3x^2-5x)e^x + e^x(6x-5)$$

$$= (3x^2+x-5)e^x$$

$$g(x) = \frac{3-2t}{5t+1} = \frac{(5t+1)\frac{d}{dt}(3-2t) - (3-2t)\frac{d(5t+1)}{dt}}{(5t-1)^2}$$

$$= \frac{(5t+1)(-2)-(3-2t)5}{(5t-1)^2}$$

$$= \frac{-10t-2-15+10t}{(5t+1)^2} = \frac{-17}{(5t+1)^2}$$



$$\frac{d}{dx}\sin x = \cos x$$

addition formular for sin(x+h)

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x(\cos h-1)}{h} + \lim_{h\to 0}\cos x\,\frac{\sin h}{h}$$

$$= \sin x\lim_{h\to 0}\underbrace{\frac{\cos h-1}{h}}_{\to 0} + \cos x\lim_{h\to 0}\underbrace{\frac{\sin h}{h}}_{=1}$$

$$= \cos x$$

two facts

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

$$\frac{d \sin x}{dx} = \cos x \qquad \frac{d \cos x}{dx} = - \sin x$$

negative

e.g. $f(x) = x^2 \sin x$

$$f'(x) = x^2 \cos x + \sin x \cdot 2x$$

$$= x^2 \cos x + 2x \sin x$$

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \quad =1 \quad = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\boxed{\frac{d}{dx} \tan x = \sec^2 x}$$

$// \frac{1}{\sin x}$

$$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$$

$// \frac{1}{\cos x}$

$$\frac{d}{dt}\sec x = \sec x \tan x$$

$$\frac{d}{dt}\cot x = -\csc^2 x$$

what x make $f'(x)=0$

$$f(x) = \frac{\sec x}{1+\tan x}$$

$$f'(x) = \frac{(1+\tan x)\sec x \tan x - \sec x \cdot \sec^2 x}{(1+\tan x)^2}$$

$$= \frac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2} = \frac{\sec x(\tan x - 1)}{(1+\tan x)^2} = 0$$

$$\tan^2 x - \sec^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{\sin^2 x - 1}{\cos^2 x}$$

$$= \frac{\sin^2 x - (\sin^2 x + \cos^2 x)}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x} = -1$$

$\sec x = \frac{1}{\cos x} \neq 0$

$\tan x - 1 = 0$

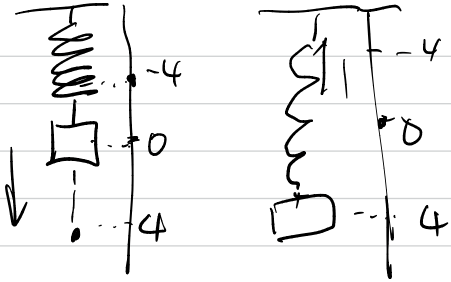
$\boxed{\tan x = 1}$

$x = \frac{\pi}{4} + n\pi$

period of $\tan x = \pi$

$\frac{f}{g} = 0 \Rightarrow f = 0$

$f \cdot g = 0 \quad f=0$ or $g=0$

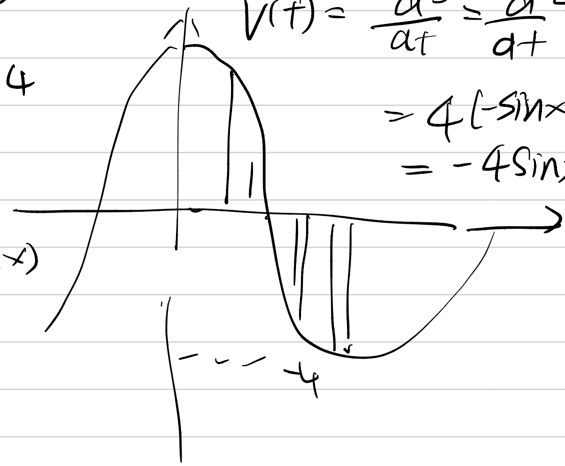


$s(t) = 4\cos t$

$$v(t) = \frac{ds}{dt} = \frac{d}{dt} 4\cos x$$

$$= 4(-\sin x)$$

$$= -4\sin x$$



$$a(t) = \frac{d\,v(t)}{dt} = \frac{d}{dt}(-4\sin x)$$

$$= -4\cos x$$

---

find 27th derivative of $\cos x$

$f(x) = \cos x$ $\quad f'(x) = -\sin$ $\quad f''(x) = -\cos x$ $\quad f''' = -(-\sin x)$ $= \sin x$

$f^{(4)} = \cos x$

$f^{(5)}(x) = f'(x)$

$f^{(6)}(x) = f''(x)$

$f^{(7)}(x) = f^{(3)}(x)$

$f^{(8)}(x) = f(x)$

$f^{(9)}(x) = f'(x)$

$\vdots$

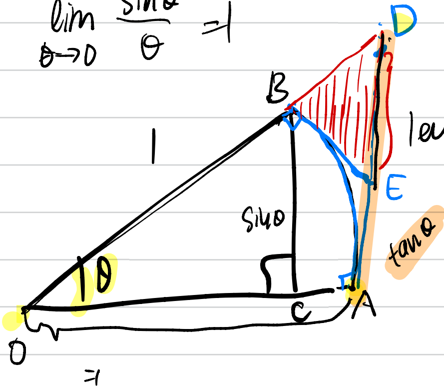
$f^{(27)}(x) = f^{23}(x) = f^{19}(x)$

$= \dots\ f^{3}(x) = \sin x$

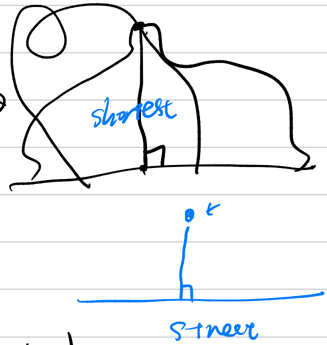
$27 = 4 \times 6 + 3$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$|DE| > |BE|$



length of arc = θ



$\text{arc } AB = \theta$

$BC = \sin\theta$

$|BC| < |\text{arc } AB|$

(assume θ, $\sin\theta > 0$)

$\sin\theta < \theta$

$\boxed{\frac{\sin\theta}{\theta} < \frac{\theta}{\theta} = 1}$

$\frac{|DA|}{|OA|} = \tan\theta$

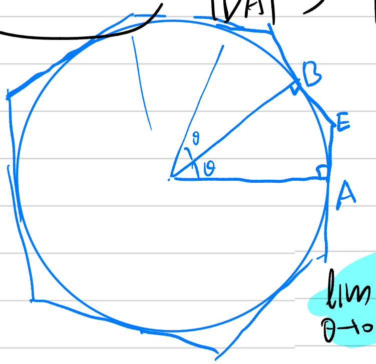
$|DA| = \tan\theta$

$(|BE| + |EA| > |\text{arc } BA|)$

$|DE| + |EA| > |BE| + |EA| > |\text{arc } BA|$

$\boxed{\frac{\sin\theta}{\theta} < 1}$

$|DA| > |\text{arc } BA|$



$\tan\theta > \theta$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\frac{\sin\theta}{\cos\theta} > \theta$ $\qquad \cos\theta > 0$

$\sin\theta > \theta\cos\theta$

$\boxed{\frac{\sin\theta}{\theta} > \cos\theta}$

$\lim_{\theta \to 0} \cos\theta \leq \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \leq 1$

$\cos 0 = 1 \leq \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \leq 1$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\Downarrow$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\Downarrow$$

$$\cos^2\theta - 1 = -\sin^2\theta$$

$$= \lim_{\theta \to 0} \frac{(\cos\theta - 1)}{\theta} \cdot \frac{(\cos\theta + 1)}{(\cos\theta + \theta)}$$

$$= \lim_{\theta \to 0} \frac{\cos^2\theta - 1^2}{\theta \cdot (\cos\theta + \theta)} = \lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta(\cos\theta + \theta)}$$

$$= \lim_{\theta \to 0} \frac{-\sin^2\theta}{\theta(\cos\theta + \theta)} = -\underbrace{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}}_{=1} \cdot \lim_{\theta \to 0} \frac{\sin\theta}{\cos\theta + \theta}$$

$$= -1 \times \left(\frac{\sin\theta}{\cos\theta + \theta}\right)$$

$$= -1 \times \left(\frac{0}{1}\right) = -1 \times 0 = 0$$

$$\frac{\sin x}{x}$$

$$f(x) = \frac{\sin 7x}{4x} = \frac{1}{4}\left(\frac{\sin 7x}{x}\right)$$

$$= \frac{1}{4}\left(\frac{7}{7}\right)\frac{\sin 7x}{x}$$

$$= \frac{7}{4}\left(\frac{\sin 7x}{7x}\right)$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{x \to 0} \frac{\sin 7x}{4x} = \frac{7}{4} \lim_{x \to 0} \frac{\sin 7x}{7x}$$

$$= \frac{7}{4} \lim_{7x \to 0} \frac{\sin 7x}{7x}$$

$\theta = 7x$

$$\sim \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$= \frac{7}{4} \times 1 = \frac{7}{4}$$

$$f(x) = x \cot x = x \frac{\cos x}{\sin x}$$

$$\lim_{x \to 0} x \cot x = \lim_{x \to 0} \frac{x \cos x}{\sin x} = \lim_{x \to 0} \frac{\frac{x \cos x}{x}}{\frac{\sin x}{x}}$$

$$= \lim_{x \to 0} \frac{\cos}{\frac{\sin x}{x}} = \frac{\lim_{x \to 0} \cos x}{\lim_{x \to 0} \frac{\sin x}{x}} = \frac{1}{1} = 1$$

Substitute we have $\cancel{\frac{0}{0}}$

$$\lim_{\theta \to \infty} \frac{\sin\theta}{\theta} \neq 1 = 0$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta \to 0} \frac{\frac{\cos\theta - 1}{\theta}}{\frac{\sin\theta}{\theta}}$$

$$= \frac{\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} = \frac{0}{1} = 0$$

$$f(x) = 3\sin x - 2\cos x$$

$$f'(x) = 3\cdot\cos x - 2(-\sin x)$$

$$= 3\cos x + 2\sin x$$

$$f(x) = x^2 + \cot x = x^2 + \frac{\cos x}{\sin x}$$

$$f'(x) = 2x + \frac{\sin x(-\sin x) - \cos x \cos x}{\sin x^2}$$

$$= 2x + \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = 2x + \frac{-1}{\sin^2 x} = 2x - \csc^2 x$$

$$h(\theta) = \theta^2 \sin\theta$$

$$h'(\theta) = \theta^2 \cos\theta + (\sin\theta)(2\theta) = \theta^2\cos\theta + 2\theta\sin\theta$$

$y = \sec\theta \tan\theta = \dfrac{\sin\theta}{\cos^3\theta}$

$= \sec\theta \cdot (\sec^2\theta) + \tan\theta(\sec\theta\tan\theta)$

$= \sec^3\theta + \sec\theta \cdot \tan^2\theta$

$= \sec\theta(\sec^2\theta + \tan^2\theta)$

$f(\theta) = (\theta - \cos\theta)\sin\theta$

$f'(\theta) = (\theta - \cos\theta)\cos\theta + \sin\theta(1 - (-\sin\theta))$

$= \theta\cos\theta - \cos^2\theta + \sin\theta + \sin^2\theta$

$f(\theta) = \theta\cos\theta\sin\theta$

$= (\theta\cos\theta)\sin\theta$

$\dfrac{d}{d\theta}(\theta\cos\theta) = \theta(-\sin\theta) + \cos\theta$

$= -\theta\sin\theta + \cos\theta$

$= (\theta\cos\theta)\cos\theta + \sin\theta \dfrac{d}{d\theta}(\theta\cos\theta)$

$= \theta\cos^2\theta + \sin\theta(-\theta\sin\theta + \cos\theta)$

$= \theta\cos^2\theta - \theta\sin^2\theta + \sin\theta\cos\theta$

passing through

$y = 2x\sin x$ at point $\left(\frac{\pi}{2}, \pi\right)$

$x = \frac{\pi}{2}$

slope $= 2$

$y - \pi = 2\left(x - \frac{\pi}{2}\right)$

$y' = 2(x\cos x + \sin x)$

$= 2x\cos x + 2\sin x$

$y'\left(\frac{\pi}{2}\right) = 2 \cdot \frac{\pi}{2} \cdot \underbrace{\cos\frac{\pi}{2}}_{=0} + 2 \cdot \sin\frac{\pi}{2}$

$= 2\sin\frac{\pi}{2} = 2$