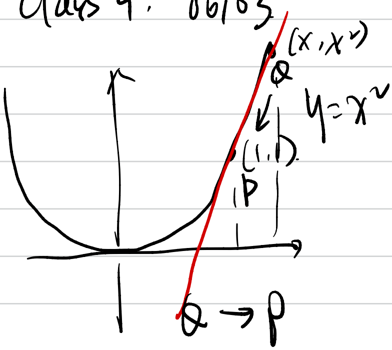
class 9. 06/03



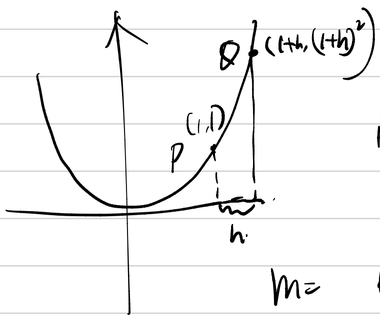
$$m_{PQ} = \frac{x^2-1}{x-1}$$

$$\lim_{x\to1}\frac{x^2-1}{x-1} = \lim_{x\to1}\frac{(x+1)(x-1)}{x-1} = \lim_{x\to1} x+1 = 2$$

Definition The tangent line to the curve $y=f(x)$ at point $(a, f(a))$ is the line through P. with slope

$$m = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$ ①

$\Downarrow$



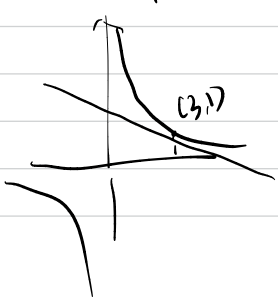
if we let $x=a+h$

②

$$m = \lim_{h\to0}\frac{f(a+h)-f(a)}{h}$$

$$y-1 = -\frac{1}{3}(x-3)$$

$f(x)=\frac{3}{x}$



$$m = \lim_{h\to0}\frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to0}\frac{\frac{3}{3+h}-1}{h} = \lim_{h\to0}\frac{\frac{3}{3+h}-\frac{3+h}{3+h}}{h} = \lim_{h\to0}\frac{\frac{-h}{3+h}}{h} = \lim_{h\to0}\frac{-1}{3+h} = \frac{-1}{3}$$

450 m hight

$$f(t) = 4.9t^2$$

find the instantaneous velocity after 5 s.

before. we find average velocity

by computing the displacement from (4.9, 5.1) divide by 0.2 (time)

(4.99. 5.01) 0.02, . . .

$$v = \lim_{h \to 0} \frac{4.9(5+h)^2 - 4.9(5)^2}{h} = \lim_{t \to 5} \frac{4.9t^2 - 4.9 \cdot 5^2}{t-5}$$

$$= \lim_{h \to 0} \frac{4.9(5^2 + \underline{2 \times 5 \cdot h} + h^2) - 4.9 \cdot 5^2}{h}$$

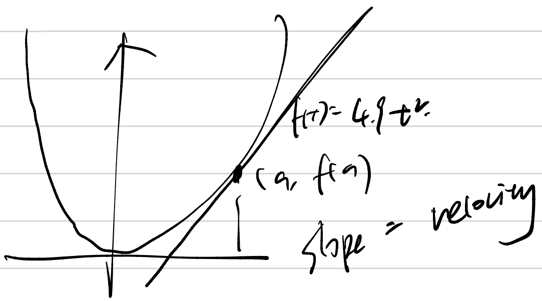
$$= \lim_{h \to 0} \frac{4.9 \times 5^2 + 4.9 \times 2 \times 5 \times h + 4.9 \times h^2 - 4.9 \times 5^2}{h}$$

$$= \lim_{h \to 0} \frac{4.9 \times 2 \times 5 \times h - 4.9 \cdot h^2}{h}$$

$$= \lim_{h \to 0} 4.9 \times 10 - 4.9 \times h$$

$$= 49 \text{ m/s}$$

Instantaneous velocity of an object with position function $f(t)$ at time $t=a$ is $v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$



Definition the derivative of a function f at a, denoted by f(a)

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ if this limit exist.

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

tangent line pass through $(2, -3)$

$f(x) = x^2 - 8x + 9$. at number 2.

$y - (-3) = -4(x-2)$

$f(2) = 2^2 - 8 \cdot 2 + 9 = 4 - 16 + 9 = -3$

$$\lim_{h \to 0} \frac{f(2+h) - (-3)}{h} = \lim_{h \to 0} \frac{(2+h)^2 - 8(2+h) + 9 + 3}{h} = \lim_{h \to 0} \frac{4 + 4h + h^2 - 16 - 8h}{h}$$

$+12$

$$= \lim_{h \to 0} \frac{h^2 - 4h}{h} = \lim_{h \to 0} h - 4 = -4$$

$f(x) = x^2 - 8x + 9$. find derivative $x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(a+h)^2 - 8(a+h) + 9 - (a^2 - 8a + 9)}{h}$$

$$= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - 8a - 8h + 9 - a^2 + 8a - 9}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2ah - 8h}{h}$$

$$= \lim_{h \to 0} h + 2a - 8 = 2a - 8$$

If we let $a = 2$ $\quad 2 \times 2 - 8 = -4$

$\boxed{f'(a) = 2a - 8} \rightarrow$ function

$\rightarrow f'(2) = -4$

$a = 2$

$$\frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}$$

$f(x) = \frac{1}{\sqrt{x}}$ derivative at number $a > 0$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a} = \lim_{x \to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x-a} = \lim_{x \to a} \frac{\frac{\sqrt{a} - \sqrt{x}}{\sqrt{x}\sqrt{a}}}{x-a}$$

$$= \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{(x-a)\sqrt{xa}} = \lim_{x - a} \frac{\sqrt{a} - \sqrt{x}}{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})\sqrt{xa}}$$

$a \in$ Domain

$$= \lim_{x \to a} \frac{-1}{(\sqrt{x} + \sqrt{a})\sqrt{xa}}$$

$$= \frac{-1}{(\sqrt{a} + \sqrt{a})\sqrt{a \cdot a}} = \frac{-1}{a \cdot 2\sqrt{a}} = \frac{-1}{2a^{3/2}}$$

$f(x)$ at point $(a, f(a))$ has slope $f'(a)$

$$\boxed{y - f(a) = f'(a)(x - a)}$$

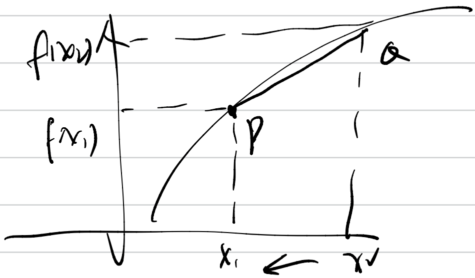
equation for tangent line

if x change from x_1 to x_2

$y = f(x)$ change of x $\quad x_2 - x_1 = \Delta x$

change of $y = f(x_2) - f(x_1) = \Delta y$

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

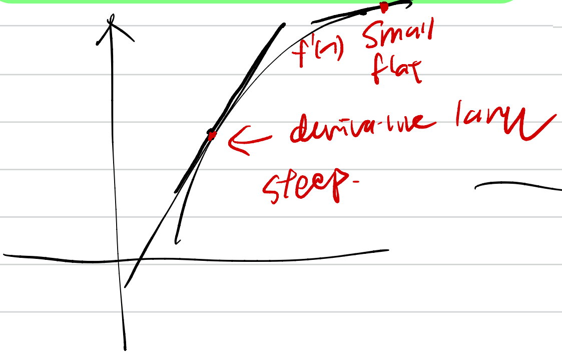


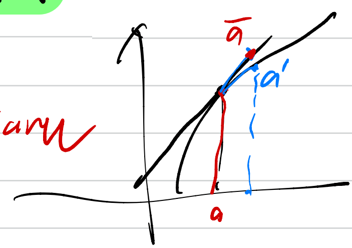
Instantaneous rate of change

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= f'(x_1)$$

The derivative $f'(a)$ is instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$





$f(t)$ position function for a. we able to find $f'(a)$

$f'(a) = v(a)$ is velocity of the particle.

speed of particle $= |f'(a)|$

$f'(a) = g(a)$ $f'(x)$ $g(x)$

$g(x)$ is function

2.8 previous $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

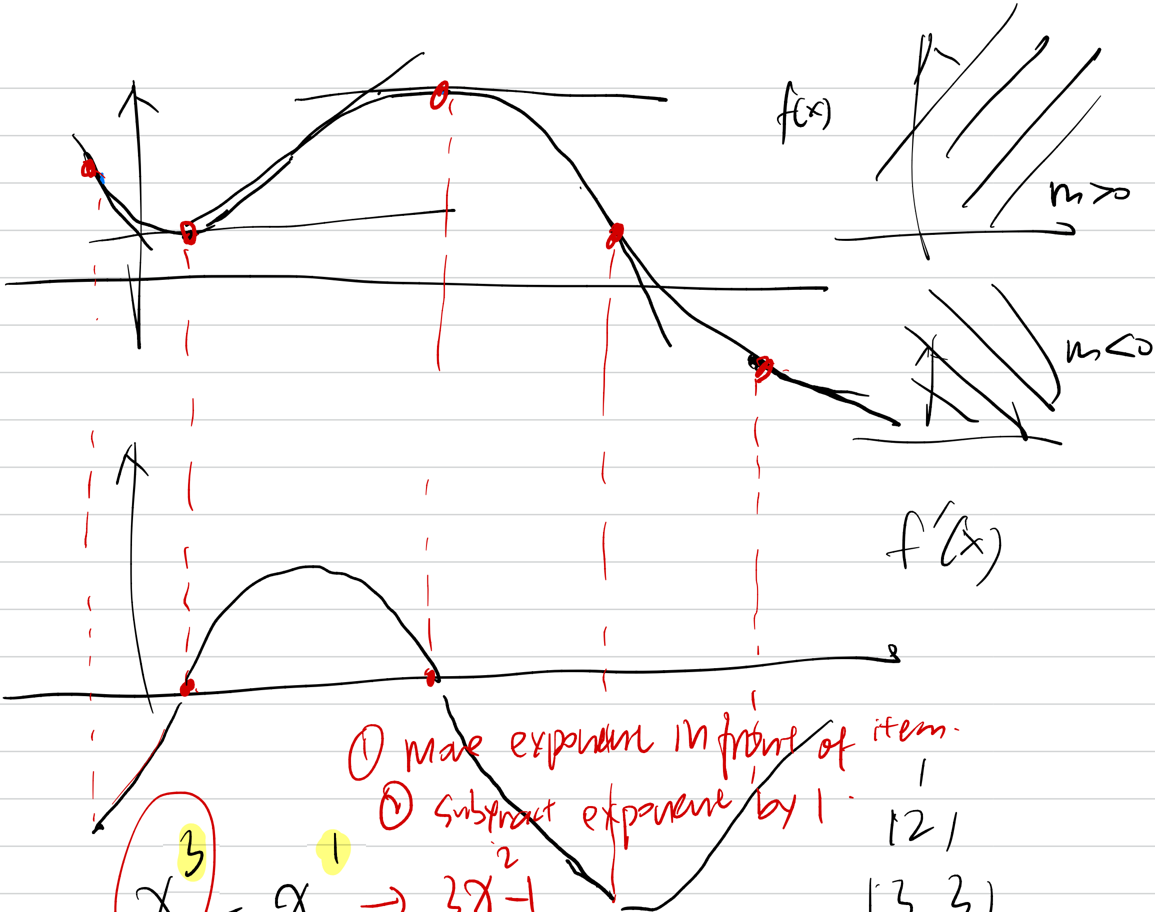
$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$X \longmapsto f'(x)$ f' is a function

derivative of f

X $f'(x)$ is the slope of tangent line of $f(x)$ at $(x, f(x))$

Domain for $f'(x) = \{x \mid f'(x) \text{ exist}\}$



① make exponent in front of item.

② subtract exponent by 1.

$x^3 - x \to 3x^2 - 1$

1

1 2 1

1 3 3 1

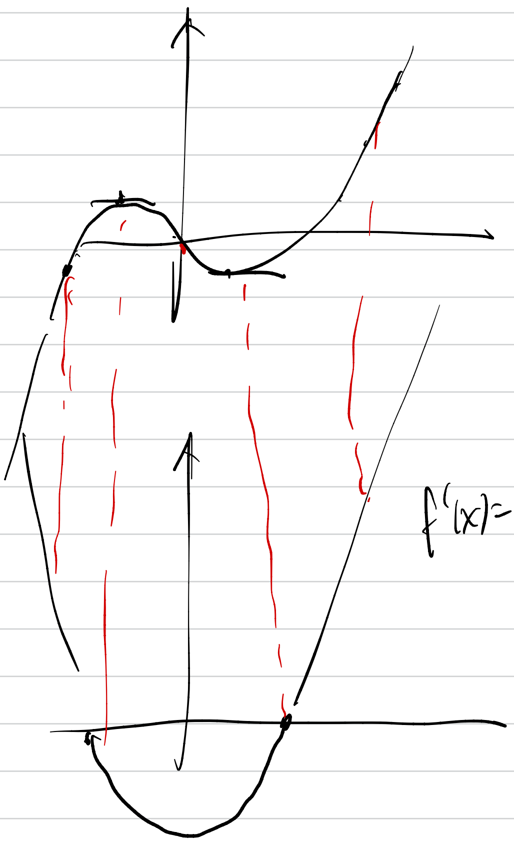
$\boxed{f(x) = x^3 - x}$ $\boxed{f'(x) = 3x^2 - 1}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3-(x+h)-(x^3-x)}{h}$$

$$= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x-h-x^3+x}{h}$$

$$= \lim_{h\to 0} 3x^2+3xh+h^2-1 = 3x^2-1$$

$f(x) = x^3 - x$



$f'(x) = 3x^2 - 1$

$[0, +\infty)$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$ verify

$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{\sqrt{x}}$$

at this case

Domain $f'(x) \subsetneq$ Domain of $f(x)$

Domain $f'(x) = (0, \infty)$

$f(x)=\frac{1-x}{2+x}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1-(x+h)}{2+(x+h)}-\frac{1-x}{2+x}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{(1-(x+h))(2+x)-(1-x)(2+x+h)}{(2+x+h)(2+x)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{(1-x-h)2+x(1-x-h)-(1-x)2-(1-x)\cdot x-(1-x)h}{(2+x+h)(2+x)}}{h}$$

$-(1-x)h$

$$=\lim_{h\to 0}\frac{(1-x)\cdot 2-h\cdot 2+x(1-x)-xh-(1-x)2-(1-x)\cdot x}{(2+x+h)(2+x)h}$$

$$=\lim_{h\to 0}\frac{-2-x-(1-x)}{(2+x+h)(2+x)}$$

$$=\frac{-2-1}{(2+x)(2+x)}=\frac{-3}{(2+x)^2}$$

$f(x)=y$

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}\big(f(x)\big) = Df(x) = D_x f(x)$$

operation

$f'(a) \longrightarrow f'(x)$

differentiation operation

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^3 - x) = 3x^2 - 1$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$y = f(x)$ $x = a$

evaluate at

$$\left.\frac{dy}{dx}\right|_{x=a}$$

A function f is differentiable at a if $f'(a)$ exist

It is differentiable on an open interval (a,b) or $(a,+\infty)$ or $(-\infty,a)$ or $(-\infty,\infty)$ if it is differentiable at every number in the interval

$f(x) = |x|$ where differentiable

where $f'(a)$ exist?

① $x > 0$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{x+h-x}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

($x+h>0$)

Since $x>0$.

h can be chosen small enough s.t. $x+h>0$

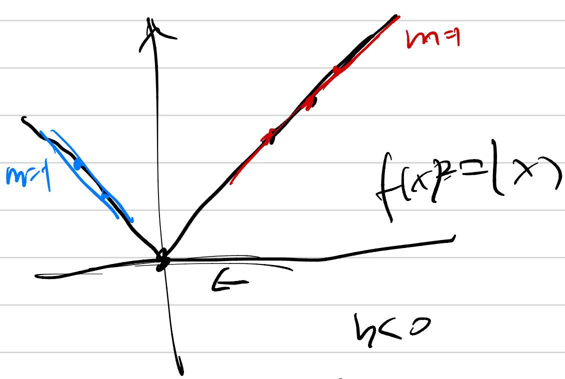
$x<0$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{-(x+h) - (-x)}{h}$$

let h small enough

$x<0 \Rightarrow x+h<0$

$$= \lim_{h\to 0} \frac{-h}{h} = -1$$



$x=0$ $\quad h>0$

$$\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{h-0}{h} = 1$$

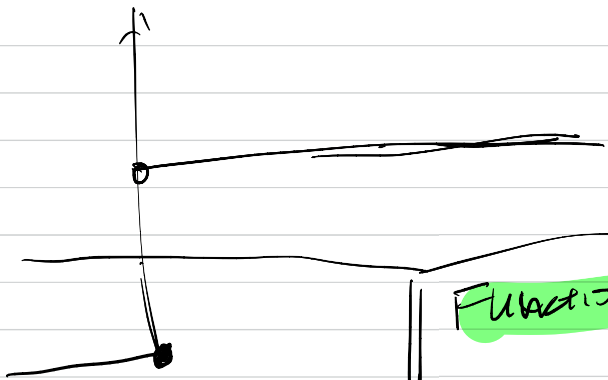
$h<0$

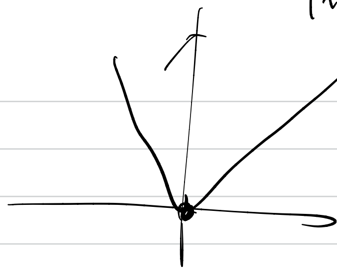
$$\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^-} \left(\frac{-h}{h}\right) = -1$$

$-1 \neq 1$ when $x=0$ $f'(0)$ DNE

Domain for $f'(x) = (-\infty, \infty) - \{0\}$





Function differentiable at a
$\Rightarrow$ function cts at a

Proof: Assume f is differentiable at a

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \text{ exist}$$

$$\lim_{x\to a} x-a=0 \text{ exist}$$

$$\underbrace{\lim_{x\to a} f(x)}_{\text{exist}} \quad \underbrace{\lim_{x\to a} g(x)}_{\text{exist}} \Rightarrow \lim_{x\to a} fg = \lim f \lim g$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}\cdot(x-a) = \underbrace{\lim_{x\to a} \frac{f(x)-f(a)}{x-a}}_{f'(a)} \underbrace{\lim_{x\to a} x-a}_{0}$$

$$\lim_{x\to a} f(x)-f(a) = f'(a)\cdot 0 = 0$$

$$\lim_{x\to a}\left(f(x)-f(a)\right) = 0 \qquad f(a) \text{ constant}$$

$$\lim_{x\to a} f(a) + 0 = \lim_{x\to a} f(a) + \lim_{x\to a}\left(f(x)-f(a)\right)$$

$$= \lim_{x\to a} f(a) + f(x) - f(a)$$

$$\boxed{f(a) = \lim_{x\to a} f(x)}$$

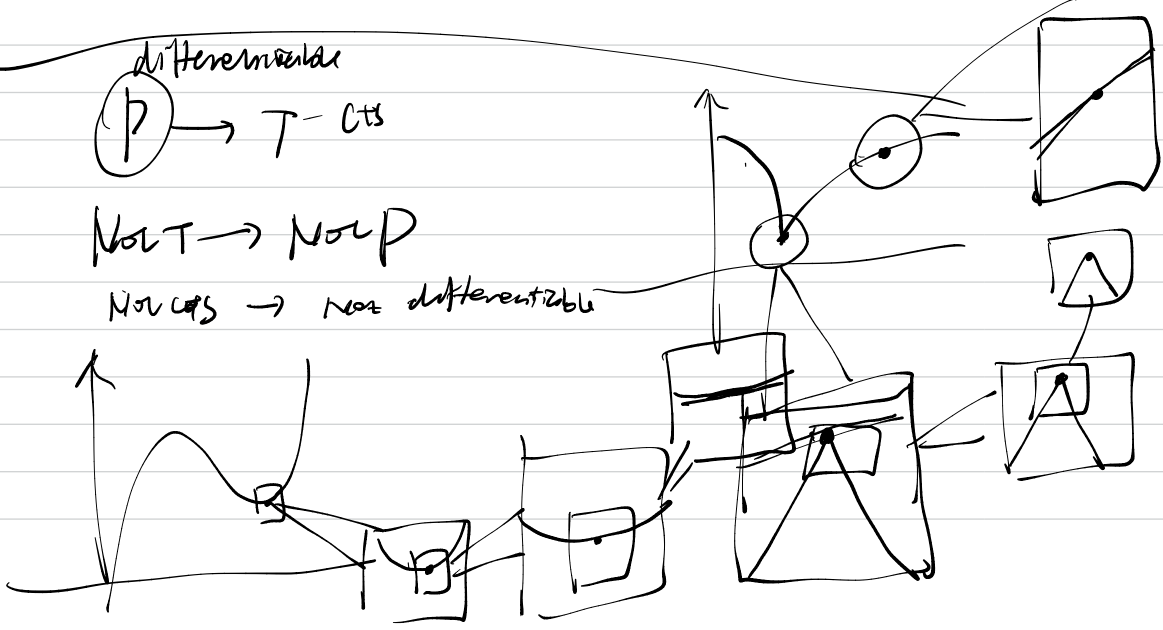
definition of cts

differentiable

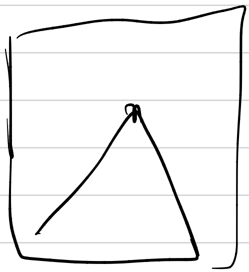
P → T - cts

Not T → Not P

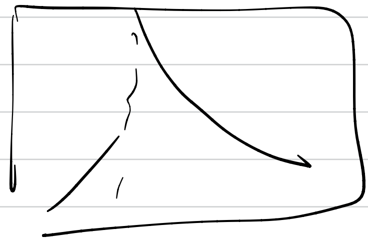
Not cts → Not differentiable



①



②



③

f

f' $\quad f''$ $\quad f''' \sim f^{(3)}$ $\quad f^{(4)}$

nth derivative

$f^{(n)}$

$f(x) = x^3 - x$ $\quad f' = 3x^2 - 1$ $\quad f''(x) = 6x$

$$f''(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 1 - (3x^2 - 1)}{h}$$

$\boxed{f'''(x) = 6}$

$f^{(4)}(x) = 0$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h} = \lim_{h \to 0} 6x + 3h$$

$= 6x$