6.17

$y = f(x)$ $\frac{dy}{dx}$ is rate of change of y
respect to x

Δy

$$\frac{dy}{dx} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Δx

$x_2 \neq x_1$ average rate of change of y respect to x

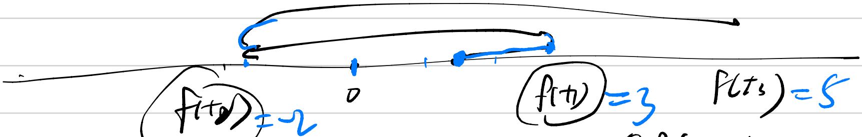
$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$ slope of line passing through

$((x_2, f(x_2)), (x_1, f(x_1)))$

If we let $x_2 \to x_1$ $\Delta x \to 0$.

$f'(x_1)$ instantaneous rate of change of y
respect x / slope of tangent line at $P(x_1, f(x_1))$

$\boxed{S = f(t)}$ position function of a particle moving in straight line



$\frac{\Delta s}{\Delta t}$ ← average velocity over Δt.

$\Delta t \to 0 \quad v = \frac{ds}{dt}$ velocity

$a = \frac{dv}{dt} = s''(t)$ acceleration.

Example $S = f(t) = t^3 - 6t^2 + 9t$

$V(t) = \frac{ds}{dt} = 3t^2 - 12t + 9$

when $t = 2$ $\quad V(2) = 3 \times 2^2 - 12 \times 2 + 9$

$= 3 \times 4 - 12 \times 2 + 9$

$= -3$

when particle at rest?

$V = 0 = 3t^2 - 12t + 9$

$3(t^2 - 4t + 3) = 0$

3 -3
1 -1

$3(t-3)(t-1) = 0$

$t = 1$ or $t = 3$

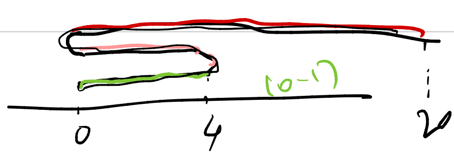
when is particle moving forward? (positive direction)

$\Rightarrow V > 0 \quad 3(t-3)(t-1) > 0 \quad \begin{cases} t > 3 \\ t > 1 \end{cases} \Rightarrow t > 3$

$\begin{cases} t-3 < 0 \\ t-1 < 0 \end{cases} \Rightarrow t < 1$ — move forward

$f(0) = 0 \quad f(1) = 1 - 6 + 9 = 4 \quad f(3) = 3^3 - 6 \times 3^2 + 9 \times 3$

$= 27 - 54 + 27 = 0$



$f(5) = 5^3 - 6 \times 5^2 + 9 \times 5 = 20$

$f(t) = t^3 - 6t^2 + 9t$

The distance traveled by particle during first 5 s.

$|f(1) - f(0)| + |f(3) - f(1)| + |f(5) - f(3)|$

forward. backward forward

$= |4-0| + |0-4| + |20-0|$

$= 4 + 4 + 20 = 28$

$f(0) = 0$

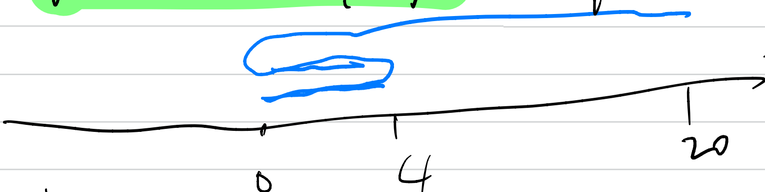
$f(1) = 4$

$f(3) = 0$

$f(5) = 20$

$v(t) = 3t^2 - 12t + 9$

$a(t) = \frac{dv}{dt} = 6t - 12$



$a(4) = 6 \times 4 - 12 = 12$

when $1 < t < 2$ $a(t) < 0$ $v(t) < 0$

when $t > 3$

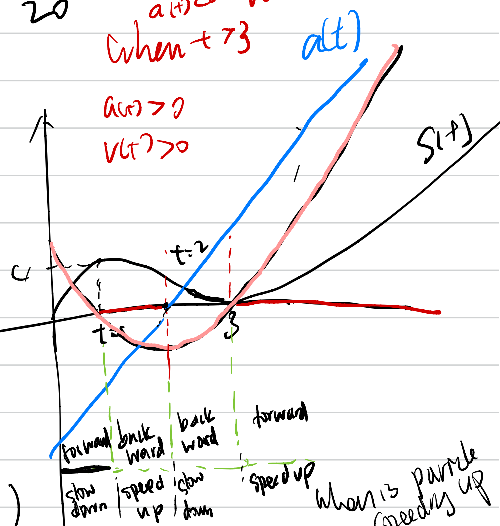
$a(t) > 0$

$v(t) > 0$

$V(t)$: when $t = 1$ or 3: $v(t) = 0$

when $t > 3$ or $t < 1$: $V(t) > 0$

when $1 < t < 3$: $V(t) < 0$



$\lim_{t_2 \to t_1} \frac{V(t_2) - V(t_1)}{t_2 - t_1} = a(t_1)$

always positive

v is positive then when $a > 0$ particle is speed up

when is particle speeding up

when $v < 0$

$a(t) = \frac{f(t_2) - f(t_1)}{t_2 - t_1} < 0$

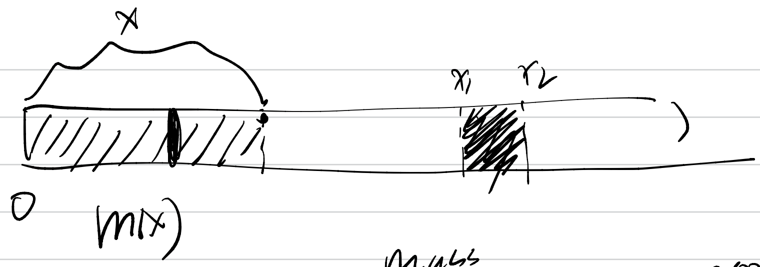
v is change from

$-3 \to -0.001$

slow down

$-3 \to -5$ speed up

when particle is speed up. $v(t)$ and $a(t)$ have same sign rod



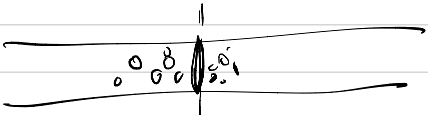
therefore we have mass function m(x) respect to length x

$$\frac{m(x_1)-m(x_2)}{x_1-x_2} = \frac{\Delta m}{\Delta x} \qquad \Delta x \to 0 \qquad \rho = \frac{dm}{dx}$$

linear density ρ the limit of average density as $\Delta x \to 0$

rate of change of mass respect to x

$$m = f(x) = \sqrt{x} \qquad \rho = \frac{dm}{dx} = \frac{1}{2\sqrt{x}} \qquad \rho(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$



plane surface

ΔQ is the net charge that passes through this surface during a time period Δt

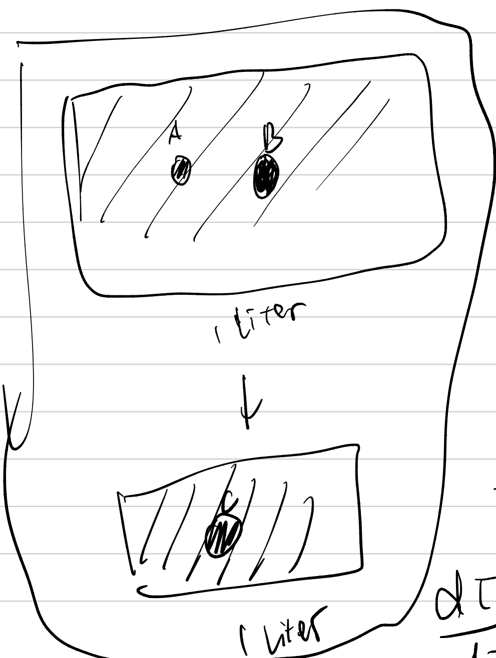
average current = $\frac{\Delta Q}{\Delta t}$

$\Delta t \to 0$ current $\frac{dQ}{dt}$

$$A + B \to C$$

reactant product

concentration of a reactant A is number of mole per liter denote by $[A]$



$$\frac{\Delta [C]}{\Delta t} = \frac{[C](t_2) - [C](t_1)}{t_2 - t_1}$$

$\Delta t \to 0$

rate of reaction = $\frac{d[C]}{dt}$

$$\frac{\Delta [A]}{\Delta t} = \frac{[A](t_2) - [A](t_1)}{t_2 - t_1}$$

$$\frac{d[C]}{dt} = -\frac{d[A]}{dt} = -\frac{d[B]}{dt}$$

$$aA + bB \rightarrow cC + dD$$

$$-\frac{1}{a}\frac{d[A]}{dt} = -\frac{1}{b}\frac{d[B]}{dt} = \frac{1}{c}\frac{d[C]}{dt} = \frac{1}{d}\frac{d[D]}{dt}$$

$n = f(t)$ animal population

$$\Delta n = \frac{f(t_2) - f(t_1)}{t_2 - t_1} \qquad \frac{\Delta n}{\Delta t} = \frac{f(t_2) - f(t_1)}{t_2 - t_1} \quad \Delta t \to 0$$

growth rate $= \frac{dn}{dt}$

$f(0) = n_0$ $\quad f(1) = 2f(0) = 2n_0$ $\quad f(2) = 2f(1) = 2^2(n_0)$

$\ldots$ $f(3) = 2^3(n_0) \ldots\ldots$ $\quad \underline{f(t) = 2^t n_0}$

$$\frac{df}{dt} = \ln 2 \cdot 2^t \cdot n_0$$

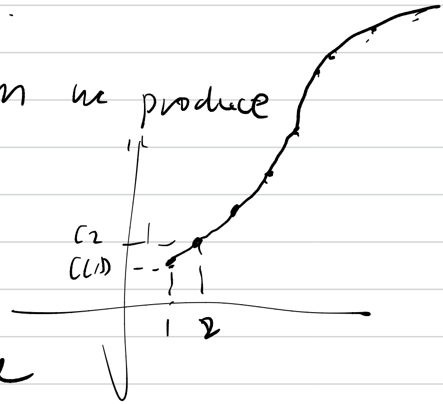
$t = 4$

$$\frac{df}{dt} = \ln 2 \cdot 2^4 \cdot n_0$$

$$= 16 \ln 2 \, n_0$$

3 clothes 5 cloth.

$C(x)$ x := # of the item we produce



$$\lim_{\Delta x \to 0} \frac{\Delta C}{\Delta x} = \frac{dc}{dx}$$

$\Delta x = 1$ let n is large
(Δx is small compare to n)

$$C'(n) \approx \Delta C = C(n+1) - C(n)$$

$$C(x) = 10{,}000 + 5x + 0.01x^2$$

$$C'(x) = 5 + 0.02x$$ 500 item

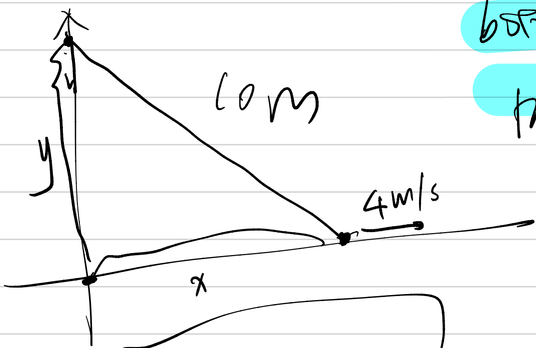
$$C'(500) = 5 + 0.02 \times 500$$
$$= 5 + 10 = \$15/\text{item}$$

$$\approx C[501] - C[500] = 15.01$$

ladder 10ft long rest against vertical wall

bottom of ladder slides away from wall at 4ft/s

How fast is the top of the ladder sliding down the wall when bottom of ladder is 6 feet from wall



$$\boxed{x^2+y^2=10^2=100}$$

x is function of t   x(t)

y is function of t   y(t)

$$\boxed{\frac{dy}{dt}\,?}$$

take implicit derivative respect to t

$$\frac{d(x^2+y^2)}{dt}=\frac{d(100)}{dt}=0$$

$$\frac{dx}{dt}=4$$

$$x=6$$

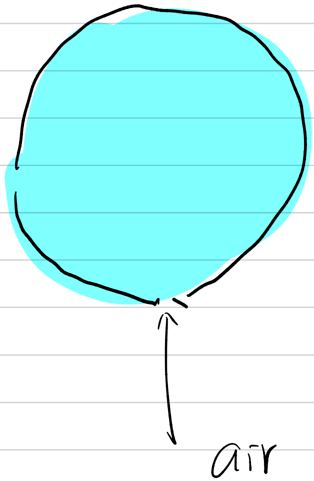
$$y=\sqrt{100-6^2}=\sqrt{100-36}=8$$

$$\boxed{2x\cdot\frac{dx}{dt}+2y\frac{dy}{dt}=0}$$

$$\underbrace{2\times6\times4}_{48}+2\times8\cdot\frac{dy}{dt}=0$$

3 feet/s

$$\frac{dy}{dt}=\frac{-48}{2\times8}=\frac{-48}{16}=-3$$



← sphere Volume $\boxed{V = \frac{4}{3}\pi r^3}$

$$\frac{dV}{dt} = 100$$

How fast is radius of balloon increasing when $d = 50$

$\Downarrow$

$r = 25$

$$\boxed{\frac{dr}{dt} ? \qquad r = 25}$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$100 = 4 \cdot \pi \times 25^2 \cdot \frac{dr}{dt}$$

$$100 = 2500\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{25\pi}$$

1. Reed the problem

2. Draw a diagram if possible

3. Assign symbol to all quantities that are function of time

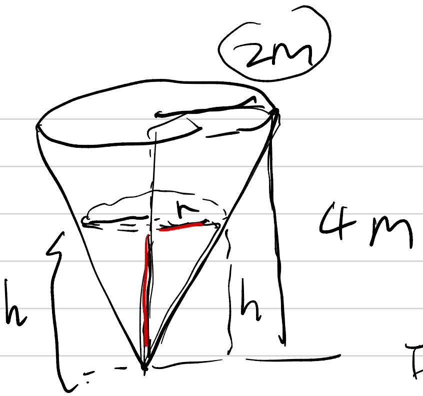
4. Express the given information by derivatives

5. Write equation that relates the various quantities of the problem

6. Using chain rule to differentiate both side respect to t

7. Substitute

unknown rate



water has been pumped into the tank at a rate $2\,m^3/min$

Find the rate where the water level is rising when water is 3 meter deep

$\frac{dV}{dt} = 2$

$\boxed{\frac{dh}{dt} \text{ when } h = 3}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{h}{r} = \frac{4}{2} \quad r = \frac{h}{2}$

$= \frac{1}{3}\pi \cdot \left(\frac{h}{2}\right)^2 \cdot h$

$\frac{dV}{dt} = \frac{d\left(\frac{\pi}{12}h^3\right)}{dt}$

$= \frac{1}{3}\pi \cdot \frac{h^3}{4}$

$\boxed{\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}}$

$V = \frac{\pi}{12}h^3$

$2 = \frac{\pi}{12} \cdot 3 \times 3^2 \cdot \frac{dh}{dt}$

$2 = \frac{27}{12}\pi \, \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{24}{27\pi} = \frac{8}{9\pi}$

car A traveling west at 50 miles/h

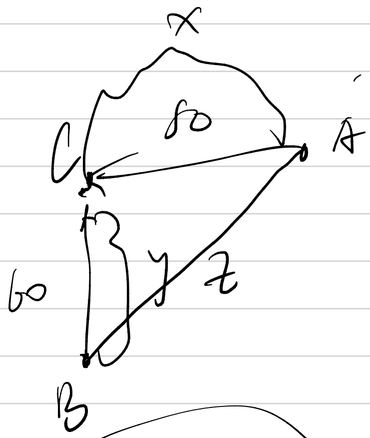
B north 60 miles/h.

what are the rate of the car approaching each other

when car A is 0.3 mile

B is 0.4 miles

from C



$$\frac{dx}{dt} = -50$$

$$\frac{dy}{dt} = -60$$

$$\frac{dz}{dt}?$$

$x = 0.3$

$y = 0.4$

$$\boxed{x^2 + y^2 = z^2}$$

$$z = 0.5 = \sqrt{0.3^2 + 0.4^2}$$

$$\frac{d(x^2+y^2)}{dt} = \frac{d(z^2)}{dt}$$

$$2x \cdot x'(t) + 2y\, y'(t) = 2z \cdot z'(t)$$

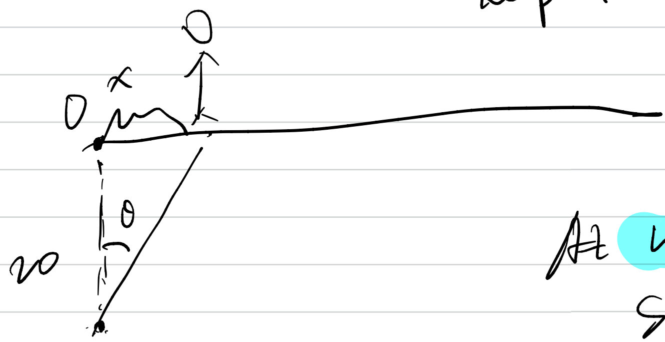
$$2 \times 0.3(-50) + 2 \times 0.4(-60) = 2 \times 0.5 \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = -78$$

a man walk along a straight path at a speed 4 ft/s

A spotlight is located on the ground 20 ft from path

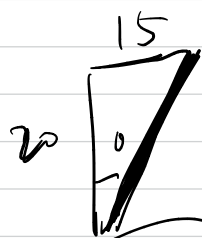
keep focus on man



At what rate is the spot light rotating when man is 15 ft from the point on the path closest to the light

$\frac{dx}{dt} = 4$

$\frac{d\theta}{dt}$? when $x = 15$



$\sqrt{15^2 + 20^2} = 25$

$\cos\theta = \frac{20}{25} = \frac{4}{5}$

$x = 20\tan\theta$

$\frac{dx}{dt} = 20 \frac{d(\tan\theta)}{dt}$

$\frac{dx}{dt} = 20\sec^2(\theta) \cdot \frac{d\theta}{dt}$

$\frac{dx}{dt} = \frac{20}{\cos^2\theta} \frac{d\theta}{dt}$

$\frac{x}{20} = \tan\theta$

$4 = \frac{20}{\left(\frac{4}{5}\right)^2} \cdot \frac{d\theta}{dt}$

$4 = \frac{125}{4} \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{16}{125}$