$\boxed{10/6/11}$

f at $g(x)$ g at x $\Rightarrow$ $(f \circ g)$ at x

diff   diff   diff   chain rule:

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(x^3-1)^{100} = 100(x^3-1)^{99}\left(\frac{d}{dx}(x^3-1)\right)$$

$$= 100(x^3-1)^{99}(3x^2)$$

$$\frac{d}{dx}(x^3-1)^{100}$$

$\boxed{u = x^3-1}$

$\boxed{y = u^{100}}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 100u^{99} \cdot (3x^2)$$

$$= 100(x^3-1)^{98}(3x^2)$$

$$\boxed{\frac{de^x}{dx} = e^x}$$

$$\frac{db^x}{dx} = \ln b \cdot (b^x)$$

$y = \sqrt{\cos x}$ $= \frac{1}{2}(\cos x)^{-\frac{1}{2}}(-\sin x) = \frac{-\sin x}{2\sqrt{\cos x}}$

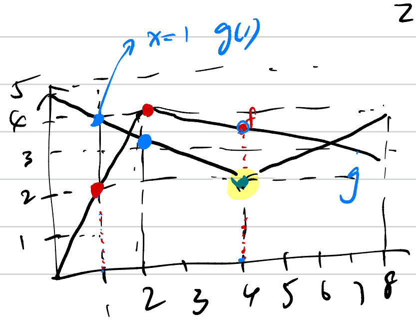
$u = \cos x$

$y(u) = \sqrt{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}}(-\sin x)$$

$$= \frac{1}{2}(\cos x)^{-\frac{1}{2}}(-\sin x)$$

$$= \frac{-\sin x}{2\sqrt{\cos x}}$$



$f \circ g'(1)$

g is diff at 1.

$\boxed{g(1) = 4}$

f is diff at 4 (g(1))

therefore f∘g is diff at 1.

$g \circ g$

when x=1 g(1) = 4

g is diff at 1.

If g is diff at g(1) = 4

x=4 ⇒ g(4) = 2

it's a corner

not diff. $(g \circ g)'(1)$ DNE

$(g \circ f)'(1)$

f(1)=2 f is diff at 1

verify

g is diff at f(1) = 2 ?

when x=2 (2, g(2)) g is diff at 2

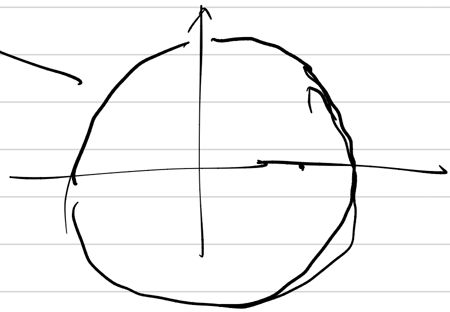
then $g \circ f$ is diff at 1.

f is function

Before $\boxed{y=f(x)}$

$\boxed{x^2+y^2=25}$

If we fix x
then y is not arbitary.



graph of
$x^2+y^2=25$

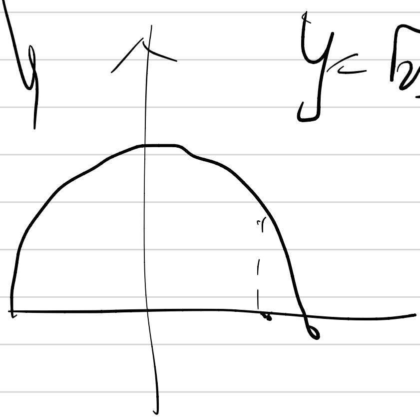
$y^2=25-x^2$

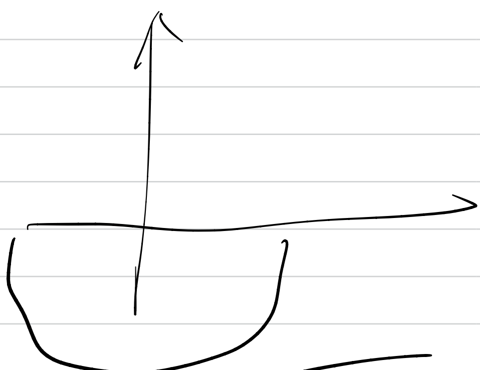
$y=\sqrt{25-x^2}$

or $y=-\sqrt{25-x^2}$

able to solve $\frac{dy}{dx}$

$y=\sqrt{25-x^2}$



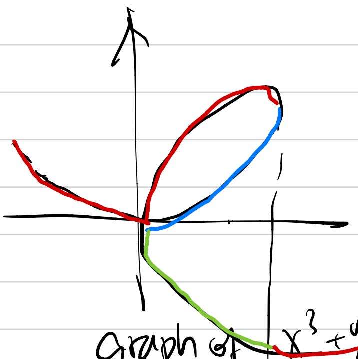


$y=-\sqrt{25-x^2}$

$$x^3+y^3=6xy$$

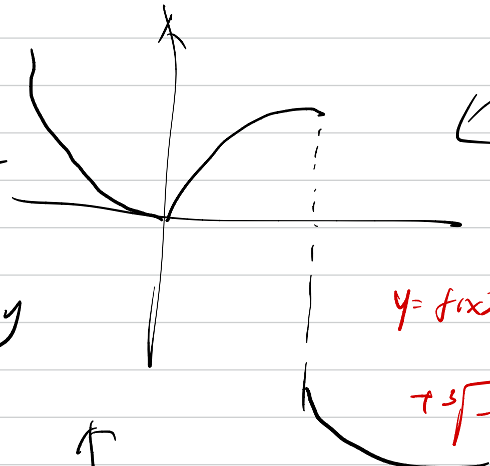
How can we have $\frac{dy}{dx}$?

hard for us to find out $y=f(x)$ explicitly



graph of $x^3+y^3=6xy$

$\doteq$

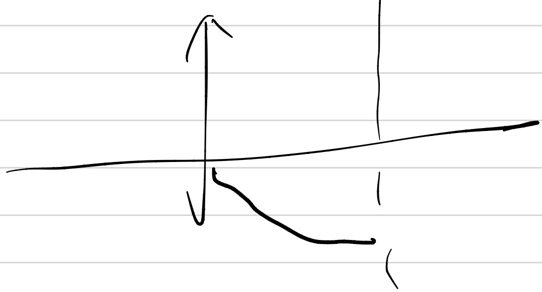


function

$$y=f(x)=\sqrt[3]{-\frac{1}{2}x^3+\sqrt{\frac{1}{4}x^6-8x^3}}+\sqrt[3]{-\frac{1}{2}x^3-\sqrt{\frac{1}{4}x^6-8x^3}}$$

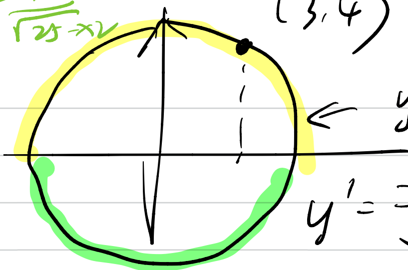
$+$

function



function

derivative function of lower circle $y' = \frac{-x}{-\sqrt{25-x^2}} = \frac{x}{\sqrt{25-x^2}}$



(3,4)

function of upper

$y = \sqrt{25-x^2}$

$y' = \frac{-x}{y}$

$= \frac{-3}{4}$

$y' = \frac{-x}{\sqrt{25-x^2}}$ upper circle

$x^3 + y^3 = 6xy$

$x^2 + y^2 = 25$

$x^2 + y^2 = 25$

$$\frac{d(x^2+y^2)}{dx} = \frac{d(25)}{dx}$$

$y^2 = 25 - x^2$

$y = \sqrt{25-x^2}$

$y = -\sqrt{25-x^2}$

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} = 0$$

y is $y(x)$

a function of x

$$2x + \frac{d(y^2(x))}{dx} = 0$$

$$2x + 2(y(x))\, y'(x) = 0$$

$$2x + 2y \cdot y' = 0$$

$$y' = \frac{-2x}{2y} = -\frac{x}{y}$$

$$x^3+y^3=6xy$$

① differentization at both side

$$\frac{d(x^3+y^3)}{dx} = \frac{d(6xy)}{dx}$$

$$\boxed{\frac{d(xy)}{dx} = xy'+y}$$

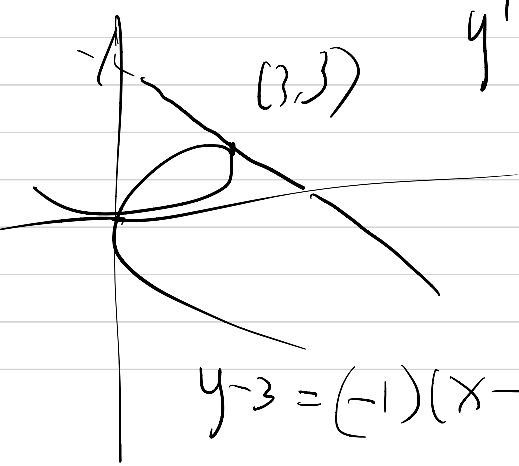
$$\frac{d(x^3)}{dx} + \frac{d(y^3)}{dx} = 6\left(\frac{d(xy)}{dx}\right)$$

$$3x^2 + 3y^2 \cdot y' = 6(xy'+y)$$

move every item contain y' to same side

$$3y^2 \cdot y' - 6x \cdot y' = 6y - 3x^2$$

$$y'(3y^2-6x) = 6y-3x^2$$



$(3,3)$

$$y' = \frac{6y-3x^2}{3y^2-6x} = \frac{2y-x^2}{y^2-2x}$$

slope

$$y'(3,3) = \frac{2\cdot 3 - 3^2}{3^2-2\cdot 3} = \frac{-3}{3} = -1$$

$$y-3=(-1)(x-3)$$

$ax^2+bx+c=0$ $\quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$ax^3+bx^2+cx+d=0$

for $\underbrace{ax^5+bx^4+cx^3+dx^2+ex+f=0}$

doesn't have general formula

Not only can't find general formula

$y^5+3x^2y^2+5x^4=12$

impossible to find expression for y interms of x

$\sin(x+y) = y^2\cos x \qquad y'$

$$\frac{d(\sin(x+y))}{dx} = \frac{d(y^2\cos x)}{dy}$$

$$\cos(x+y)\left(\frac{d}{dx}(x+y)\right) = y^2(-\sin x) + \cos x \cdot \left(\frac{d}{dx}y^2\right)$$

$$\boxed{\cos(x+y)(1+y') = y^2(-\sin x) + \cos x\,(2y \cdot y')}$$

$$\cos(x+y) + y'\cos(x+y) = -y^2\sin x + \cos x \cdot 2y \cdot y'$$

$$\cos(x+y) + y^2\sin x = \cos x \cdot 2y \cdot y' - \cos(x+y)\cdot y'$$

$$\cos(x+y) + y^2\sin x = y'(2y\cos x - \cos(x+y))$$

$$y' = \frac{\cos(x+y) + y^2\sin x}{2y\cos x - \cos(x+y)}$$

$$\boxed{x^2+xy = x+3\sin y}$$

$(1,0)$

Still need to find y'

$$\frac{d(x^2+xy)}{dx} = \frac{d(x+3\sin y)}{dx}$$

take derivative at both side

$$\frac{d(x^2)}{dx} + \frac{d(xy)}{dx} = \frac{d(x)}{dx} + \frac{d(3\sin y)}{dx}$$

compute

$$2x + (xy' + y\cdot 1) = 1 + 3\cos(y)\cdot y'$$

move y' to one side

$$2x + y - 1 = 3\cos y \cdot y' - xy'$$

$$2x+y-1 = y'(3\cos y - x)$$

divide coefficient to get y'

$$y' = \frac{2x+y-1}{3\cos y - x}$$

equation

$$(y-0) = \frac{1}{2}(x-1)$$

$$y'(1,0) = \frac{2\times 1 + 0 - 1}{3\times\cos 0 - 1} = \frac{1}{2}$$

slope $= \frac{1}{2}$

$y^3 + ye^x = 10 \qquad y'$

$$\frac{d(y^3+ye^x)}{dx} = \frac{d(10)}{dx}$$

$$\frac{d(y^3)}{dx} + \frac{d(ye^x)}{dx} = 0 \quad 3y^2 \cdot y' + ye^x + e^x y' = 0$$

$$(3y^2+e^x)\, y' = -ye^x$$

$$y' = \frac{-ye^x}{3y^2+e^x}$$

$\sin(x+y) = \cos x + \cos y$

$$\frac{d \sin(x+y)}{dx} = \frac{d(\cos x + \cos y)}{dx}$$

$$\cos(x+y)(1+y') = -\sin x - \sin(y) \cdot y'$$

$$\cos(x+y) + \cos(x+y)y' = -\sin x - (\sin y) \cdot y'$$

$$\cos(x+y)\, y' + \sin y \cdot y' = -\sin x - \cos(x+y)$$

$$y'(\cos(x+y) + \sin y) = -\sin x - \cos(x+y)$$

$$y' = \frac{-\sin x - \cos(x+y)}{\cos(x+y) + \sin y}$$

$\boxed{x^2+4y^2=4}$ $\quad y'$ $\quad y''$?

first method

$$y' = \frac{-2x}{8y}$$

$$\frac{d(x^2+4y^2)}{dx} = \frac{d4}{dx}$$

$$2x + 4\cdot 2\cdot y\cdot y' = 0$$

$$\boxed{2x + 8yy' = 0}$$

$$\boxed{y' = \frac{-2x}{8y}}$$

$$y'' = \frac{8y(-2) - (-2x)8y'}{(8y)^2}$$

Second method

$$= \frac{-16y + 16x\cdot y'}{64y^2}$$

$$\frac{d(2x+8yy')}{dx} = 0$$

$$= \frac{-y + xy'}{4y^2}$$

$$= \frac{-y + x\left(\frac{-2x}{8y}\right)}{4y^2}$$

$$2 + \frac{8d(yy')}{dx} = 0$$

$$2 + 8(y\cdot y'' + y'y') = 0$$

$$= \frac{-8y^2 - 2x^2}{32y^3}$$

$$8(y\cdot y'' + (y')^2) = -2$$

$$yy'' + (y')^2 = \frac{-1}{4}$$

$$y'' = \frac{-\frac{1}{4} - (y')^2}{y} = \frac{-\frac{1}{4} - \left(\frac{2x}{8y}\right)^2}{y}$$

$xy + e^y = e$ value of y'' at point $x=0$

$$\frac{d(xy + e^y)}{dx} = \frac{d(e)}{dx}$$

$$y'' = \frac{(x+e^y)(-y') - (-y)(1+e^y \cdot y')}{(x+e^y)^2}$$

$$\frac{d(xy)}{dx} + \frac{d(e^y)}{dx} = 0$$

find value of y''

$$(xy' + y) + e^y \cdot y' = 0$$

When $x=0$

$$0 \times y + e^y = e$$

$$(x+e^y)\, y' = -y$$

$$e^y = e$$

$$y' = \frac{-y}{x+e^y}$$

$$y = 1$$

$$y'(0,1) = \frac{-1}{0+e^1} = \frac{-1}{e}$$

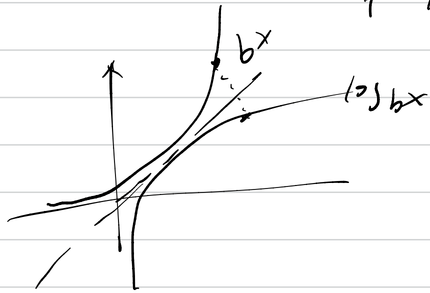
$$y''(0,1) = \frac{(0+e)\left(\frac{1}{e}\right) + (1)\left(1+e\left(-\frac{1}{e}\right)\right)}{(0+e)^2}$$

$$= \frac{1}{e^2} = \frac{1}{e^2}$$

inverse function

$b^x \sim \log_b x$

if f is one-to-one diff funct
f^{-1} is one to one diff function



f is differe every where

f^{-1} is also diff everywhere

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

when $b = e$ $\ln e = 1$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$y = \log_b x$

y'?

by Implicit differention

$\Downarrow$

$b^y = x$

y' ✓ !!

$$\frac{d(b^y)}{dx} = \frac{dx}{dx} \Rightarrow (b^y \cdot \ln b)\, y' = 1$$

$$y' = \frac{1}{b^y \ln b}$$

$$= \frac{1}{x \cdot \ln b}$$

$$f = \log_b x \qquad f' = \frac{1}{x \ln b}$$

$$f = \ln x \qquad f' = \frac{1}{x}$$

$y = \ln(x^3+1) \qquad y'$

$$y' = \frac{1}{(x^3+1)} \cdot (3x^2) = \frac{3x^2}{x^3+1}$$

$y = \ln(\sin x) \qquad y'$

$$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

$$\frac{d}{dx}\left(\ln\left(\frac{x+1}{\sqrt{x-2}}\right)\right) = \frac{1}{\frac{x+1}{\sqrt{x-2}}} \cdot \frac{d}{dx}\left(\frac{x+1}{\sqrt{x-2}}\right)$$

$$= \frac{\sqrt{x-2}}{x+1}\left(\frac{\sqrt{x-2} \cdot 1 - \frac{(x+1)}{2\sqrt{x-2}}}{(x-2)}\right)$$

$$= \frac{(x-2) - \frac{x+1}{2}}{(x+1)(x-2)} = \frac{2x-4-x-1}{2(x+1)(x-2)}$$

$$= \frac{x-5}{2(x+1)(x-2)}$$

$$\frac{d}{dx}\left(\ln\frac{(x+1)}{\sqrt{x-2}}\right)$$

$$= \frac{d}{dx}\left(\ln (x+1) - \ln\sqrt{x-2}\right)$$

$$= \frac{d}{dx}\left(\ln (x+1)\right) - \frac{d}{dx}\left(\ln\sqrt{x-2}\right)$$

$$= \frac{1\times 1}{x+1} - \frac{1}{\sqrt{x-2}}\cdot\frac{1}{2\sqrt{x-2}}$$

$$= \frac{1}{x+1} - \frac{1}{2(x-2)} = \frac{2x-4-x-1}{2(x+1)(x-2)} = \frac{x-5}{2(x+1)(x-2)}$$