MATH 141 Final Exam

May 1, 2023

NAME (please print legibly): _____

University ID Number: _____

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

Directions

- Enter your answers where indicated in order to receive credit.
- Show your work. Unjustified answers will **not** receive credit.
- Calculators and notes are not permitted.
- If you are confused about the wording of a question or need clarification, raise your hand and **ask a proctor** about it.

Part A

1. (10 points) Find the solution(s) of the equation

$$e^{2x} - 10e^x + 21 = 0.$$

Answer:

2. (12 points)

(a) Evaluate $\sin(\tan^{-1}(-\sqrt{3}))$.

Answer:

(b) Suppose $\pi \leq \theta \leq 2\pi$ and $\cos\theta = \dfrac{1}{2}$. Compute $\csc(\theta)$.

Answer:

3. (16 points) Find the vertical and horizontal asymptote(s) of the function

$$f(x) = \frac{x^3 + x^2 - 6x}{2x(x-1)(x+3)}.$$

Vertical asymptote(s):	Horizontal asymptote(s):

4. (15 points) Evaluate the following limits.

(a) $\lim_{h\to 0} \dfrac{\tan\left(\frac{3\pi}{4}+h\right)-\tan\left(\frac{3\pi}{4}\right)}{h}$

Answer:

(b) $\lim_{x\to\infty} \dfrac{\cos(3x)}{x^2}$

Answer:

(c) $\lim_{x\to\infty} \cos\left(\dfrac{3}{x}\right)$

Answer:

5. (12 points) Suppose $f(x) = 2x^2 - 7$ and $g(x)$ has tangent line $y = 3x + 4$ at $x = 1$.

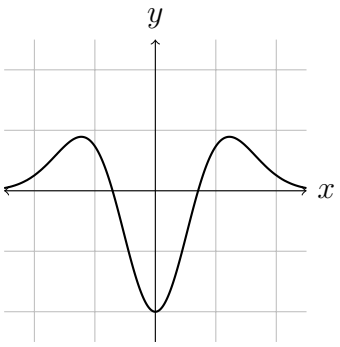
(a) Find $g'(1)$.

Answer:

(b) Let $h(x) = f(g(x))$. Find $h'(1)$.

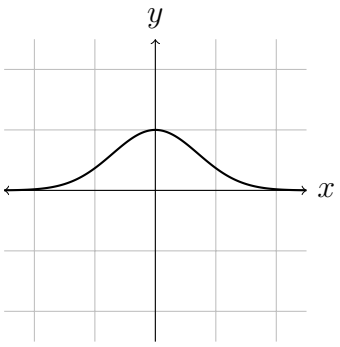
Answer:

6. (8 points) Below are the graphs of a function f, its first derivative f', and its second derivative f''. Identify which graph is of which function.



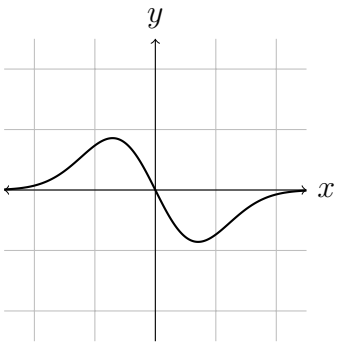
Circle answer:

f	f'	f''



Circle answer:

f	f'	f''



Circle answer:

f	f'	f''

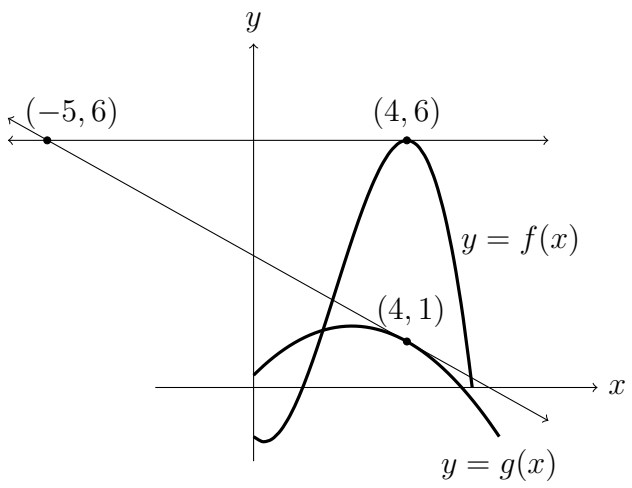
7. (15 points) Differentiate the following functions. Circle or box your final answer.

(a) $f(x) = 6^x \ln(7x)$

(b) $g(t) = \dfrac{e^{3t^2}}{\sin(-4t) + \sqrt{t}}$

(c) $h(z) = \sqrt[4]{z^5 + \cos(2z^3)}$

8. (12 points) The differentiable functions f and g are graphed below along with their tangent lines at $x = 4$.



(a) Find $f'(4)$.

Answer:

(b) Find $g'(4)$.

Answer:

(c) Let $h(x) = \dfrac{f(x)}{g(x)}$. Find $h'(4)$.

Answer:

Part B

9. (15 points) Differentiate the following functions. Circle or box your final answers. (Hint: You may need/want to use **logarithmic differentiation** for one or more of these.)

(a) $f(x) = \arctan\left(\sqrt{2x}\right)$

(b) $g(x) = x^{\tan(x)}$

(c) $r(x) = \dfrac{(x^3 - x)^3 \sqrt[3]{\cos(x)}}{e^{3x}(x^2 - 4)^4}$

10. (12 points) Use implicit differentiation to find the equation of the tangent line of

$$x^2y + 3y^2x = 4$$

at the point $(1, 1)$.

Answer:

11. (9 points) Use linear approximation to estimate the value of $\sqrt{15.8}$.

Answer:

12. (12 points) Suppose the **velocity** of a particle moving the along the real number line is $v(t) = t^2 - 8t + 15$.

(a) Determine when the particle is moving to the left between $t = 0$ and $t = 10$.

Answer:

(b) Determine when the particle is slowing down between $t = 0$ and $t = 10$.

Answer:

13. (12 points) Find the absolute minimum and maximum of

$$f(t) = t - 6\sqrt{t+2}$$

in $[2, 23]$.

Absolute maximum:	Absolute minimum:

14. (12 points) Suppose the legs of a right triangle have length 3 cm and 5 cm. Suppose the area of the triangle is increasing at a rate of 2 cm^2/s and the length of the short leg is decreasing at a rate of 1 cm/s.

(a) At what rate is the length of the long leg increasing?

Answer:

(b) At what rate is the length of the hypotenuse increasing?

Answer:

15. (12 points) Evaluate the following limits.

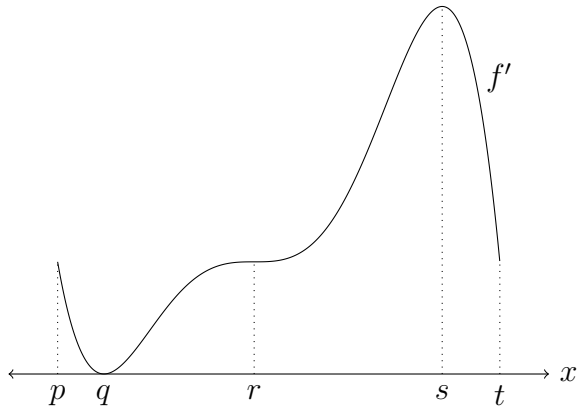
(a) $\lim_{x \to 0} \dfrac{e^x - e^{-2x}}{x^2}$

Answer:

(b) $\lim_{x \to 0^+} x^{2x}$

Answer:

16. (16 points) Let f be a differentiable function. Below is the graph of its **derivative** f'. Answer the following questions about f (**not** f').



(a) Find the critical point(s) of f on the interval $[p, t]$. Classify the point(s) as relative minima, relative maxima, or neither.

Answer:

(b) Determine where f is concave up on the interval $[p, t]$.

Answer:

(c) Find the inflection point(s) of f on the interval $[p, t]$.

Answer:

(d) List the values of $f(p)$, $f(r)$, and $f(t)$ from least to greatest.

Answer: _____ < _____ < _____

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