MATH 141 Midterm 2

November 15, 2022

NAME (please print legibly): _____

Your University ID Number: _____

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: _____

Enter your answers where indicated in order to receive credit. Calculators and notes are not permitted. If you are confused about the wording of a question or need a clarification, you should raise your hand and **ask a proctor** about it.

Unless otherwise indicated, you must show all work to justify your answers and receive full credit.

1. (15 points)

Let
$$f(x) = \frac{2x^k + x^3 - 2}{3x^4 - 2x}.$$

(a) Find all values of k so that $\lim\limits_{x\to\infty} f(x) = 0$, or explain why this is not possible.

Answer:

(b) Find all values of k so that $\lim\limits_{x\to\infty} f(x) = \infty$, or explain why this is not possible.

Answer:

(c) Find all values of k so that $\lim\limits_{x\to\infty} f(x) = \frac{2}{3}$, or explain why this is not possible.

Answer:

2. (10 points) Suppose we use the following limit to determine the derivative of a function $f(x)$ at $x = a$.

$$\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}.$$

(a) What is the function $f(x)$ and the number a?

Answer:

(b) Find the derivative $f'(a)$ using any method you wish.

Answer:

3. (20 points) Suppose that the functions f and g satisfy the following:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	$-\pi$	-2	2	3
2	1	7	5	4

(a) Let $h(x) = 3f(x) - 3g(x) + 3$. Find $h'(2)$.

Answer:

(b) Let $h(x) = \dfrac{f(x)}{g(x)}$. Find $h'(2)$.

Answer:

(c) Let $h(x) = (g \circ f)(x)$. Find $h'(2)$.

Answer:

(d) Let $h(x) = e^{f(x)g(x)}$. Find $h'(2)$.

Answer:

4

4. (10 points) Determine the derivative of the following functions. You do not have to simplify. Circle your final answer.

(a) $f(x) = e^x \ln(x) + \tan(x)$

(b) $h(z) = 2^z + z^{\sqrt{2}} + e^2 + 2^\pi$

5. (20 points) Determine the derivatives of the following functions. You do not have to simplify. Circle your final answer.

(a) $f(x) = 5e^{x^3} + \ln(\ln x)$

(b) $g(t) = \ln(te^{-2t})$

(c) $h(z) = (5z^2 - 6z)^9(z^3 + 7)$

(d) $k(w) = \cos(\sqrt{w^5 + 3w^2})$

6. (15 points) Determine an equation for the line tangent to the curve satisfying

$$x^2 + xy = x + 3\sin(y)$$

at the point $(1,0)$.

Answer:

7. (10 points) Let $g(x) = (x^2 + 1)^x$. Find $g'(x)$. (Hint: Use logarithmic differentiation.)

Answer: