Math 141: Calculus I

Final Exam

December 16, 2014

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Indicate your instructor with a check in the appropriate box:

Prof. Kalyani Madhu	MWF 09:00 – 09:50 AM	
Prof. Alex Rice	TR 2:00 – 3:15 PM	
Prof. Saul Lubkin	MW 2:00 – 3:15 PM	
Prof. Evan Dummit	TR 4:50 – 6:05 PM	

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **You are responsible for checking that this exam has all 19 pages.**

Part A		
QUESTION	VALUE	SCORE
1	16	
2	12	
3	10	
4	10	
5	10	
6	20	
7	12	
8	10	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
9	18	
10	15	
11	12	
12	15	
13	20	
14	20	
TOTAL	100	

Part A

1. (16 points)

(a) Find all real numbers x such that

$$2x^2 + x - 3 \geq 0.$$

(b) Find the domain of the function:

$$f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$$

(c) Find $f \circ g \circ h$ where

$$f(x) = \tan(x), \quad g(x) = \frac{x}{x-1}, \quad \text{and} \quad h(x) = \sqrt[3]{x}.$$

2. (12 points) Find the exact value of:

(a) $\log_4 \frac{1}{8}$

(b) $3^{-2\log_3 5}$

(c) $\cos(3\arctan(-1))$

3. (10 points) Let $f(x) = \dfrac{4x^2 - 1}{x^2 - 4}$.

(a) Find all horizontal asymptotes of the graph of $y = f(x)$.

(b) Find all vertical asymptotes of the graph of $y = f(x)$.

4. (10 points) Let $r(x) = \begin{cases} -x + c & \text{for } x < -1 \\ x^2 + 2 & \text{for } -1 \leq x < 0 \\ 0 & \text{for } x = 0 \\ x^3 - 4x + 2 & \text{for } x > 0 \end{cases}$.

(a) Determine, WITH JUSTIFICATION, whether $r(x)$ is continuous at $x = 0$.

(b) Find c such that r is continuous at $x = -1$.

5. (10 points) Consider the function $f(x) = \dfrac{1}{x-4}$,

(a) Use the limit definition of the derivative to compute $f'(x)$. (You MUST use the definition and show work. Other methods will receive no credit.)

(b) Find an equation for the line tangent to the graph of $y = f(x)$ at $x = 6$.

6. (20 points) Compute the following derivatives.

(a) $g'(t)$ if $g(t) = 4t^3 - \dfrac{10}{\sqrt[5]{t}} - e^{\pi}$.

(b) $\dfrac{d}{dx}\left(\dfrac{e^{\cos(x)}}{x^7 - 1}\right)$

(c) $\dfrac{dR}{ds}$ if $R = \sqrt{s}\ln(s^2+4)$

(d) $h'(x)$ if $h(x) = \sin^{-1}(e^{1/x})$.

7. (12 points) Use logarithmic differentiation to compute the following derivatives.

(a) $f'(x)$ if $f(x) = (\tan(x))^{8x^5+1}$

(b) $\dfrac{d}{dt}\Big(\dfrac{\sqrt{t}(t^2+3)}{(t+1)(t-5)^7}\Big)$

8. (10 points)

(a) Find $f''(x)$ if $f(x) = \sec(7x)$.

(b) Suppose the curve $\mathcal{C}$ is defined implicitly by the equation $x^3y + 5xy^2 = 22$. Find an equation for the line tangent to $\mathcal{C}$ through the point $(1, 2)$.

Part B

9. (18 points) At time $t = 0$ seconds, I throw a potato off the roof of the Hylan building, located on Mars. Its position at time t is equal to $p(t) = -2t^2 + 6t + 36$ meters above ground, where $t \geq 0$.

(a) When does the potato reach its maximum height? What is its velocity at this time?

(b) How long does it take the potato to hit the ground?

(c) How fast is the potato moving when it hits the ground?

10. (15 points) A spherical snowball has an initial radius of 6 cm. It melts so that the radius of the snowball decreases at a rate of 0.2 cm/min. What is the rate of change of the volume of the snowball 5 minutes later? (Hint: The volume of a sphere is given by $V = \dfrac{4\pi r^3}{3}$.)

11. (12 points) Let $g(x) = e^x$.

(a) Find the linearization of $g(x)$ at $a = 0$.

(b) Use the linearization in part (a) to approximate $e^{-0.01}$.

12. (15 points) Let $g(x) = x^3 - 6x^2 + 9x$.

(a) Find all critical numbers of $g(x)$.

(b) Find the minimum and maximum values of $g(x)$ on the interval $[-1, 2]$.

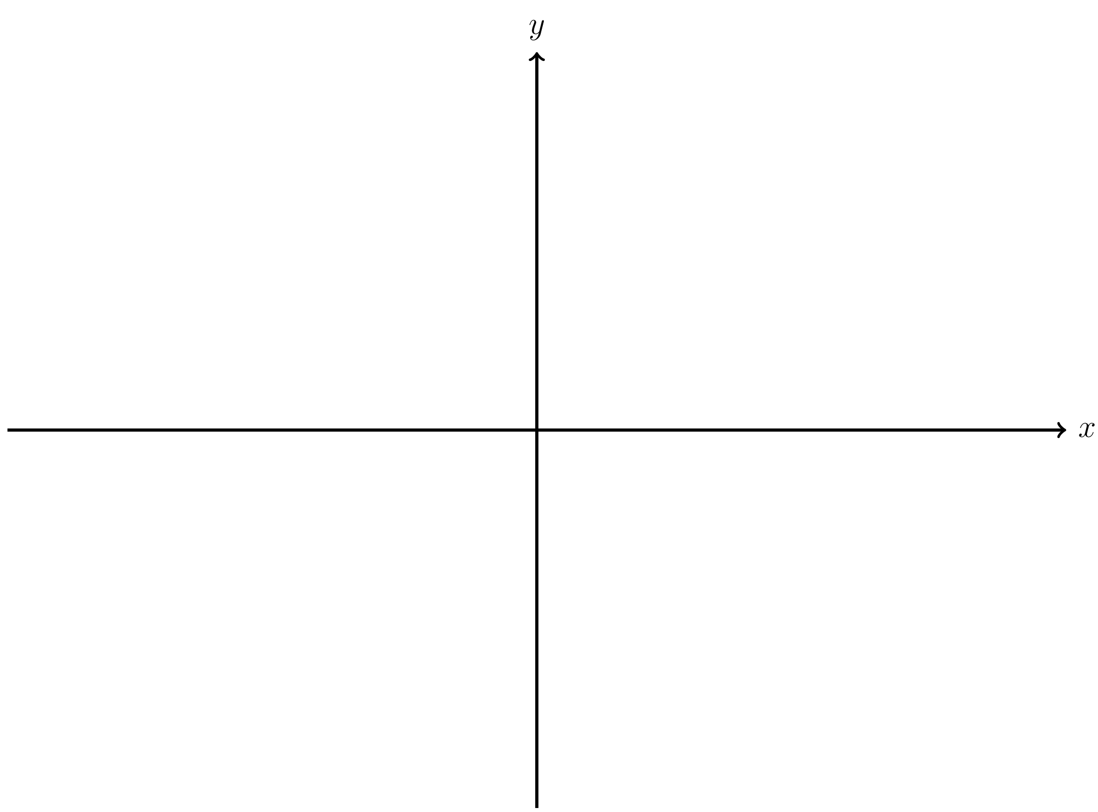
13. (20 points) Consider the function $f(x) = 4x^3 - x^4$

(a) On what intervals is f decreasing?. On what intervals is f increasing?

(b) On what intervals is the graph $y = f(x)$ concave upward? On what intervals is $y = f(x)$ concave downward?

(c) Find all local maximum and minimum values of f. Find all points of inflection of $y = f(x)$. (Recall, $f(x) = 4x^3 - x^4$.)

(d) Sketch a graph of f on the axes below. Label x- and y-intercepts. Label points at which f has local minimum or maximum values. Label points of inflection.



14. (20 points) Find the following limits:

(a) $\lim_{x\to\infty} \dfrac{\sqrt{9x^4+4x^2}}{\sqrt{4x^4+9x^2}}$.

(b) $\lim_{s\to-\infty} \dfrac{\sqrt{9s^2+4s}}{4s+9}$

(c) $\lim_{t \to 0} \frac{e^t - t - 1}{7t^2}$

(d) $\lim_{x \to \infty} \left(\frac{\ln(2x-3)}{3x+4} \right)$